CALCULUS 3 EXAM 2 Spring 2006 NAME_____

REC. INSTR._____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK

(15) 1. a) For the curve $y = x^3/3$ in the xy-plane, find the curvature as a function of x.

b) Suppose an object is moving along the curve in part a) from left to right. It is moving in such a way that its speed as a function of the time t is

$$\text{speed} = t^2 \text{ ft/sec.}$$

Suppose also that at time $t = 2$ seconds the object is at the point $(x,y) = (1,1/3)$. Find a_T, a_N and the acceleration vector <u>when t = 2 seconds</u>.

$a_T =$

$a_N =$

$a =$

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(15) 2. Suppose that a quantity Q depends upon x and y according to

$$Q = x \exp(xy^2 - 10) .$$

Both x and y are changing with the time t and at a certain instant you know that $x = 3$, $y = 2$, $dx/dt = 2$ and $dy/dt = -1$.

Use the chain rule to find dQ/dt at this instant.

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(20) 3. Let $F(x,y,z) = x^2z + y^2 + xyz^2$.

a) Find the gradient vector field of $F(x,y,z)$.

b) Find the directional derivative of $F(x,y,z)$ at the point $(x,y,z) = (3,2,1)$ in the direction of the vector $\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.

c) Find the equation of the plane which is tangent to the surface $F(x,y,z) = 19$ at the point $(x,y,z) = (3,2,1)$.

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(15) 4. Use the method of Lagrange Multipliers to find the largest value and the smallest value of $f(x,y) = x^2 + 2y^2 - 4y$ on the circle $x^2 + y^2 = 9$.

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(20) 5. Let $f(x,y) = 3x^2y - 2x^3 + 2y^3 - 24y$. Find all the critical points of $f(x,y)$ and then use the second partials test to classify each critical point.

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(15) 6. Use a double integral to calculate the volume of the 3D-region which is under the paraboloid $z = x^2 + y^2$ and above the region in the first quadrant of the xy-plane that is bounded by the lines $y = x$, $x = 2$, and $y = 0$.