Properties of Laplace transforms

(A) Transformation of derivatives:

$$\mathcal{L}\{f^{(n)}(t)\} = s^n\,F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0);$$

in particular,

$$\mathcal{L}\{f'(t)\} = s\,F(s) - f(0)\ , \qquad \mathcal{L}\{f''(t)\} = s^2\,F(s) - sf(0) - f'(0).$$

(B) Transformation of integrals: $\mathcal{L}\left\{\displaystyle\int_0^t f(\tau)\,d\tau\right\} = \frac{F(s)}{s}$.

(C) Translation on the s-axis: $\mathcal{L}\{e^{at}\,f(t)\} = F(s-a)$.

(D) Translation on the t-axis: $\mathcal{L}\{u(t-a)\,f(t-a)\} = e^{-as}\,F(s)$.

(E) Definition of convolution: $(f*g)(t) := \displaystyle\int_0^t f(\tau)\,g(t-\tau)\,d\tau$.

(F) The convolution property: $\mathcal{L}\{f*g\} = F(s)\,G(s)$.

(G) Differentiation of transforms: $\mathcal{L}\{(-t)^n f(t)\} = F^{(n)}(s)$.

(H) LT of a periodic function $f(t)$ of period p: $F(s) = \dfrac{1}{1-e^{-ps}}\displaystyle\int_0^p e^{-st}\,f(t)\,dt$.

Table of Laplace transforms

$f(t)$	$F(s)$
1	$\dfrac{1}{s}\ (s>0)$
$t^n\ \ (n>0,\text{ integer})$	$\dfrac{n!}{s^{n+1}}\ \ (s>0)$
$t^a\ \ (a>-1)$	$\dfrac{\Gamma(a+1)}{s^{a+1}}\ \ (s>0)$
e^{at}	$\dfrac{1}{s-a}\ \ (s>a)$
$\cos kt$	$\dfrac{s}{s^2+k^2}\ \ (s>0)$
$\sin kt$	$\dfrac{k}{s^2+k^2}\ \ (s>0)$
$\cosh kt$	$\dfrac{s}{s^2-k^2}\ \ (s>\lvert k\rvert)$
$\sinh kt$	$\dfrac{k}{s^2-k^2}\ \ (s>\lvert k\rvert)$
$u_a(t) = u(t-a)\quad (a\ge 0)$	$\dfrac{e^{-as}}{s}\ \ (s>0)$
$\delta_a(t) = \delta(t-a)\quad (a\ge 0)$	$e^{-as}\ \ (s>0)$