

Here is an idea: write

$$\begin{aligned}
 f'(a) &= \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \\
 &= \lim_{x \rightarrow a} \frac{x^{3/2} - a^{3/2}}{x - a} \\
 &= \lim_{x \rightarrow a} \left(\frac{x^{3/2} - a^{3/2}}{x - a} \cdot \frac{x^{3/2} + a^{3/2}}{x^{3/2} + a^{3/2}} \right) \\
 &= \lim_{x \rightarrow a} \frac{x^3 - a^3}{(x - a)(x^{3/2} + a^{3/2})} ,
 \end{aligned}$$

and then use the formula

$$x^n - a^n = (x - a) (x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \cdots + x^2a^{n-3} + xa^{n-2} + a^{n-1}) .$$

Alternatively, you can use the same formula to write

$$x^{3/2} - a^{3/2} = (\sqrt{x})^3 - (\sqrt{a})^3 = (\sqrt{x} - \sqrt{a}) [(\sqrt{x})^2 + \sqrt{x}\sqrt{a} + (\sqrt{a})^2]$$

and

$$x - a = (\sqrt{x})^2 - (\sqrt{a})^2 = (\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a}) .$$

Do not forget to state the domain of the function and the domain of its derivative!