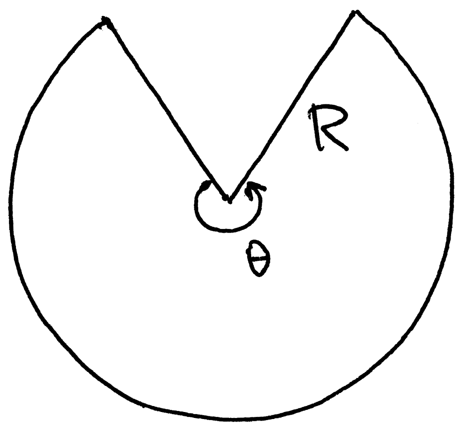
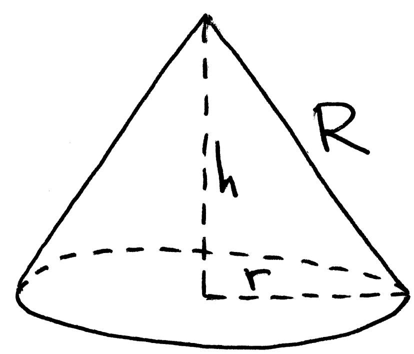
Hint to Problem 3.7/41

Let the angle of the sector that is cut out be $(2\pi - \theta)$ radians. Then the angle at the origin of the remaining sector will be θ radians (see the figure on the left). The length of the





curvilinear part of the boundary of the remaining sector is $\frac{\theta}{2\pi}$ times the circumference of a circle of radius R, i.e., it is equal to $\frac{\theta}{2\pi}\,2\pi R$. This length is equal to the circumference of the base of the conical drinking cup, therefore we can find the radius r of the base of the cone from

$$\frac{\theta}{2\pi}\,2\pi R = 2\pi r \ .$$

The height h of the cone can be found easily in terms of R and θ by using the Pythagorean Theorem (see the figure on the right). Once r and h are known (in terms of R and θ), the volume of the cone is

$$(\text{volume of the cone}) = \frac{1}{3}\,(\text{area of the base})\,(\text{height}) = \frac{1}{3}\pi r^2 h \ .$$

It is convenient to introduce the variable

$$x := \frac{\theta}{2\pi}$$

and to write the volume as a function of this new variable, i.e., to define the function

$$f(x) := (\text{volume of the cone}) \ ,$$

where the volume of the cone is expressed in terms of the new variable x (and the radius R, which is constant).

You have to find the value x^* of x at which the function f has an extremum. You will find only one such value, which is clearly a maximum, since V is non-negative and it is zero when $\theta = 0$ (i.e., when $x = 0$) and when $\theta = 2\pi$ (i.e., when $x = 1$).