Hints to Problems 5.2/61 and 5.3/46

The torus is obtained by rotating about the y-axis the circle in the (x, y)-plane centered at $(R, 0)$ and with radius r (i.e., described by the equation $(x - R)^2 + y^2 = r^2$).

By the method of **slicing** (by planes perpendicular to the y-axis), the volume is given by the integral

$$\pi \int_{-r}^{r} \left[\left(R + \sqrt{r^2 - y^2} \right)^2 - \left(R - \sqrt{r^2 - y^2} \right)^2 \right] dy \ .$$

To obtain this expression, consider the circle $(x - R)^2 + y^2 = r^2$ in the (x, y)-plane as made up of the graphs of two functions, $x = f(y)$ and $x = g(y)$, the "outside" one given by $f(y) = R + \sqrt{r^2 - y^2}$, and the "inside" one given by $g(y) = R - \sqrt{r^2 - y^2}$ (to obtain these expressions, express x in terms of y from the equation $(x - R)^2 + y^2 = r^2$).

By the method of **cylindrical shells**, the volume is given by the integral

$$\int_{R-r}^{R+r} 2\pi x \cdot 2\sqrt{r^2 - (x - R)^2} \, dx \ .$$