MATH 3413 Problems assigned on 4/3/14

Sec. 9.3: problems 18, 20.

Hint for Problem 9.3/18: This problem illustrates the dangers in differentiating a Fourier series termwise (i.e., term by term). Differentiate the Fourier series of t^2 on $t \in (0, 2)$ given in the problem term by term. Compare your result with the Fourier series of the function $2t$ on $t \in (0, 2)$ which can be easily obtained from your result in Problem 9.2/17. Discuss your result. Which condition from Theorem 1 on page 601 was violated?

Sec. 9.4: problem 1.

Hint: The Fourier series for $F(t)$ can be easily found from the result in Example 1 of Section 9.1 (page 585).

Additional problem 1. Using the properties of the Laplace transform, one can show that the solution of the initial value problem

$$\begin{aligned} x'(t) + 3x(t) &= f(t) \\ x(0) &= 0 \end{aligned} \tag{1}$$

can be written in the form

$$x(t) = \int_0^t f(\tau)\, \mathrm{e}^{3(\tau - t)}\, \mathrm{d}\tau \ . \tag{2}$$

In this problem you will check that the function $x(t)$ defined in (2) indeed satisfies the initial value problem (1). You will need to use the following formula:

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_{\phi(t)}^{\psi(t)} F(\tau, t)\, \mathrm{d}\tau = F(\psi(t), t)\, \psi'(t) - F(\phi(t), t)\, \phi'(t) + \int_{\phi(t)}^{\psi(t)} \frac{\partial F(\tau, t)}{\partial t}\, \mathrm{d}\tau \ . \tag{3}$$

(a) Show that $x(t)$ given by (2) satisfies the initial condition in (1).

(b) If you represent $x(t)$ from (2) in the form $\int_{\phi(t)}^{\psi(t)} F(\tau, t)\, \mathrm{d}\tau$, then write explicitly the functions $F(\tau, t)$, $\phi(t)$, and $\psi(t)$.

(c) Find explicitly $F(\psi(t), t)\, \psi'(t)$ and $F(\phi(t), t)\, \phi'(t)$.

(d) Find explicitly $\int_{\phi(t)}^{\psi(t)} \frac{\partial F(\tau, t)}{\partial t}\, \mathrm{d}\tau$.

(e) Using your results from (b), (c), and (d), prove that $x(t)$ given by (2) satisfies the differential equation in (1).