MATH 3113 – Homework assigned on 9/4/13

In all problems of this homework assume that the arguments of the logarithms are positive. Of course, in real life one needs to be more careful!

Problem 1. Find the general solution of the ODE

$$(y^2 - xy)\,dx + (x^2 + xy)\,dy = 0$$

using each of the following methods.

(a) First notice that it can be written as a homogeneous equation, and solve it using the standard substitution for homogeneous equations.

(b) The way the equation in this problem is written, it is *not* exact. However, if you multiply it by $\frac{1}{xy^2}$, it becomes exact – do this and solve it as an exact equation. (Honestly, I found the factor $\frac{1}{xy^2}$ by first solving the equation in its original form as a homogeneous equation. In real life, there is seldom anybody to tell you what to do with an equation to make it exact...)

Problem 2.
So far you have learned how to solve several types of first order equations: separable, linear, Bernoulli, homogeneous, exact, and a second-order ODE with a missing y. Each of the following two equations can be solved by using two of these methods. Solve each of the equations below using two different methods, and show that the solutions obtained by using different methods are the same.

(a) $(3x^2 + 2y^2)\,dx + 4xy\,dy = 0;$

(b) $\dfrac{dy}{dx} = \sqrt{y}\,\cot x - y\,\cot x.$

Problem 3.
Solve the following differential equations.

(a) $xy'' + y' = 4x$

(b) $xy^{(4)} = 1$

Hint: One can solve integrals of the form $\int z^n \ln z\,dz$ (for n nonnegative integer) by setting $u := \frac{1}{n+1} z^{n+1}$ (hence $du = z^n\,dz$), $v = \ln z$, and integrating by parts.

(c) $y''' = -(y'')^2$