Additional problem assigned on 9/26/2017

Additional problem. Consider the function defined implicitly by the relation

$$x^3 - y^3 = 7 \ . \tag{1}$$

(a) Prove that the point $(x^*, y^*) = (2, 1)$ belongs to the curve described by the equation (1).

(b) Use implicit differentiation to find the derivative $\dfrac{dy}{dx}$ as a function of x and y.

(c) Compute the value of $\left.\dfrac{dy}{dx}\right|_{(2,1)}$.

(d) Write down the equation of the tangent line to the curve described by the equation (1) at the point $(2, 1)$. Figure 1 shows the curve and the tangent line to it at the point $(2, 1)$.

(e) Use implicit differentiation again to show that the second derivative $\frac{d^2y}{dx^2}$ as a function of x and y is given by

$$\frac{d^2y}{dx^2} = \frac{2x}{y^2}\left(1 - \frac{x^3}{y^3}\right) = \frac{2x}{y^2}\,\frac{y^3 - x^3}{y^3} = -\frac{14x}{y^5} \ .$$

What did you use at the last step in this chain of equalities?

(f) Find the value $\left.\dfrac{d^2y}{dx^2}\right|_{(2,1)}$ of the second derivative at the point $(2, 1)$.

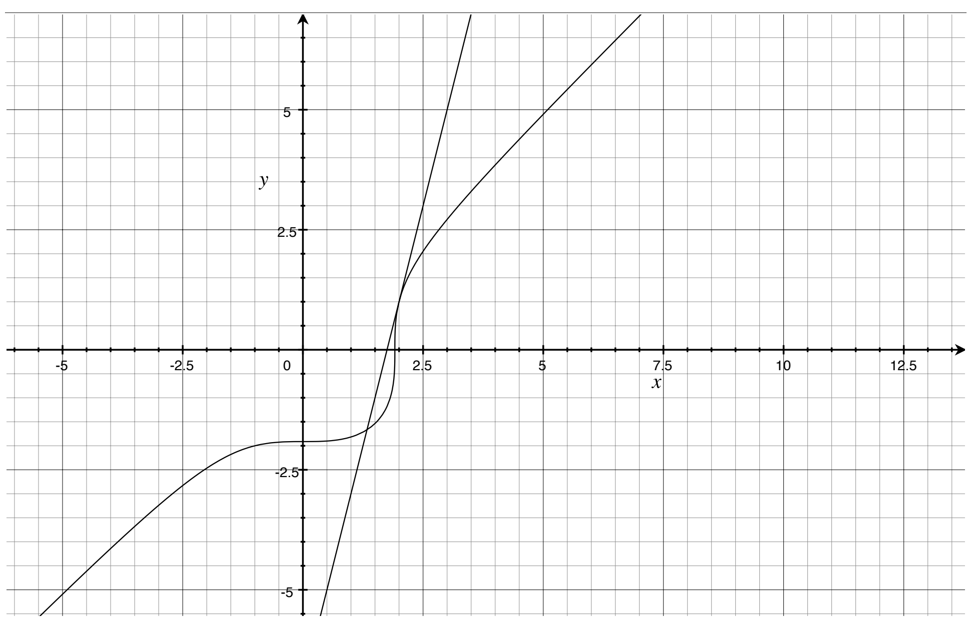

Figure 1: The curve described by equation (1) and the tangent line to it at the point $(2, 1)$.