MATH 3113 – Homework assigned on 10/28/13

Sec. 7.6: Problem 5 (do not plot the solution, just derive the expression for $x(t)$ and look at its graph in the back of the book).

Hint: See Problem 1 from the homework assigned on 10/25/13.

Additional problem 1. Let $f : \mathbb{R} \to \mathbb{R}$ be a function that is differentiable infinitely many times. Recall that the nth derivative, $\delta_a^{(n)}(t) := \frac{\mathrm{d}^n}{\mathrm{d}t^n}\delta_a(t)$, of $\delta_a(t)$ is defined by the formula

$$\int_{\mathbb{R}} \delta_a^{(n)}(t)\, f(t)\, \mathrm{d}t := (-1)^n\, f^{(n)}(a)\ . \tag{1}$$

The motivation for this definition came from treating the derivatives of $\delta_a(t)$ as ordinary functions, integrating by parts, and using that at $\pm\infty$ the function $\delta_a(t)$ is "equal" to zero. In this problem you will give a meaning of the formal definition of a derivative of $\delta_a(t)$ that looks like the derivative of an "ordinary" function:

$$\frac{\widetilde{\mathrm{d}}}{\mathrm{d}t}\,\delta_a(t) \quad \text{“}:=\text{''} \quad \lim_{h\to 0} \frac{\delta_a(t+h)-\delta_a(t)}{h}\ ; \tag{2}$$

here the tilde over the derivative sign simply means that this definition is different from the definition (1) of the derivative of $\delta_a(t)$. Inspired by (2), define

$$\int_{\mathbb{R}} \left(\frac{\widetilde{\mathrm{d}}}{\mathrm{d}t}\,\delta_a(t)\right) f(t)\, \mathrm{d}t := \lim_{h\to 0} \int_{\mathbb{R}} \frac{\delta_a(t+h)-\delta_a(t)}{h} f(t)\, \mathrm{d}t\ . \tag{3}$$

(a) Change the variable t in $\int_{\mathbb{R}} \delta_a(t+h)\, f(t)\, \mathrm{d}t$ to $z = t + h$ to compute this integral.

(b) Using your result from part (a), find $\displaystyle\int_{\mathbb{R}} \frac{\delta_a(t+h)-\delta_a(t)}{h} f(t)\, \mathrm{d}t$.

(c) Find $\displaystyle\int_{\mathbb{R}} \left(\frac{\widetilde{\mathrm{d}}}{\mathrm{d}t}\,\delta_a(t)\right) f(t)\, \mathrm{d}t$ defined by (3), and compare your result with $\int_{\mathbb{R}} \delta_a'(t)\, f(t)\, \mathrm{d}t$ given by equation (1). Discuss briefly your findings.