MATH 2924 – Additional problem assigned on 11/09/15

Additional problem.

Infinite products are defined as

$$\prod_{n=1}^{\infty} b_n := b_1 \, b_2 \, b_3 \, \cdots \, .$$

Similarly to the definition for convergence of an infinite sum, we say that the infinite product $\prod_{n=1}^{\infty} b_n$ *converges* if the sequence $\{\pi_n\}_{n=1}^{\infty}$ defined by

$$\pi_n := \prod_{j=1}^{n} b_j$$

converges. In this problem you will find a criterion for convergence of an infinite product.

(a) Use the Limit Comparison Test to prove that if $a_n > 0$ for all $n \in \mathbb{N}$, then the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} \ln(1 + a_n)$ are either both convergent or both divergent.

(b) Use your result in part (a) to prove that if $a_n > 0$ for all $n \in \mathbb{N}$, then the series $\sum_{n=1}^{\infty} \ln(1 + a_n)$ and the infinite product $\prod_{n=1}^{\infty} (1 + a_n)$ are either both convergent or both divergent.