# MATH 2924 – Additional problem assigned on 11/11/15

**Additional problem 2.** (Additional Problem 1 was assigned on 11/09/15.)

(a) Prove by induction the identity

$$\prod_{j=1}^{n} \cos\frac{x}{2^j} = \frac{1}{2^n}\,\frac{\sin x}{\sin\dfrac{x}{2^n}}\ ,$$

where $x \neq 2^r m\pi$, for any integers $r$ and $m$ (simply to avoid the case where the sine in the denominator becomes zero for some $n$).

(b) Use the identity proved in part (a) to find the infinite product

$$\prod_{j=1}^{\infty} \cos\frac{x}{2^j}\ .$$

(c) Use the trigonometric identity for cosine of half angle (or, equivalently, for cosine of double angle), to prove by induction that

$$\cos\frac{\pi}{2^{n+1}} = \frac{1}{2}\ \underbrace{\sqrt{2+\sqrt{2+\cdots+\sqrt{2}}}}_{n \text{ square roots}}\ .$$

(d) Prove the famous formula of François Viète (published in 1593)

$$\frac{\pi}{2} = \frac{2}{\sqrt{2}}\cdot\frac{2}{\sqrt{2+\sqrt{2}}}\cdot\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2}}}}\cdots\ ,$$

which can be used to compute the value of $\pi$ with any desired accuracy!!! The rate of convergence of Viète's formula is not very high, but one can apply tricks to accelerate the convergence – see, e.g., the paper by Rick Kreminski "$\pi$ to thousands of digits from Vieta's formula" *Mathematics Magazine*, Vol. 81, no. 3 (2008), pp. 201–207.