MATH 2924 – Additional FFT problem assigned on 11/11/15

Additional FFT Problem. On December 28, 2013, Shigeru Kondo used Alexander Yee announced that they have calculated 12,100,000,000,000 digits of π. They used the a program called y-cruncher, developed by Yee, and performed their computations on a single desktop computer built by Kondo; the computation took 94 days (between 10:15 p.m. on September 25, 2013 and 10:23 p.m. on December 28, 2013, Japan Standard Time) – see

`http://www.numberworld.org/misc_runs/pi-12t/`

`http://www.numberworld.org/y-cruncher/`

In their computations Kondo and Yee used the following formula derived by the brothers David and Gregory Chudnovsky, who relied on some ideas of the famous Inidian mathematician Srinivasa Ramanujan (1887–1920):

$$\frac{1}{\pi} = \frac{\sqrt{10005}}{4270934400} \sum_{k=0}^{\infty} \frac{(-1)^k (6k)!}{(k!)^3\,(3k)!} \, \frac{13591409 + 545140134k}{640320^{3k}} \, .$$

In this problem you will use Mathematica to find the rate of convergence of the right-hand side of this formula to the exact value of $\frac{1}{\pi}$. You can define the function `chud[n]` which computes the sum of the first `n` terms of Chudnovsky's formula:

```
termPi[k_]=(-1)^k*(6*k)!/(k!)^3/(3*k)!*(13591409+545140134*k)/640320^(3*k)

chud[n_]=Sqrt[10005]/4270934400*Sum[termPi[k], {k, 0, n}]
```

After you type each line in Mathematica, press SHIFT, hold it down, and press RETURN. The underscores after `k` and `n` in `termPi[k_]` and `chud[n_]` tell Mathematica that we are defining new functions, and `k` and `n` the variables of these functions.

To find the numerical value with accuracy of 1000 digits of the difference between the exact value of $\dfrac{1}{\pi}$ and the partial sum of the sum containing, say, 8 terms – which in our notations will be equal to `chud[7]` – you can type the following:

```
N[chud[7] - 1/Pi, 1000]
```

There will a problem, however, and Mathematica will complain that its internal precision limit is not enough for the computation (try it!). That is why you have to type

```
Block[{$MaxExtraPrecision = 1000}, N[chud[7] - 1/Pi, 1000]]
```

(a) Compute the numerical values of the absolute error $E_n = \left|\frac{1}{\pi} - \texttt{chud}[n]\right|$ for $n = 0, 1, 2, 3, 4, 5, 6, 7$, and write your results in a table (there is no need to write more than 3–4 digits of accuracy of E_n in the table).

(b) For the values of n used in part (a), show that your numerical results give $\frac{E_{n+1}}{E_n} \approx 10^{-14}$. Can you express E_n approximately in terms of E_0? I do not want anything sophisticated, just a VERY ROUGH approximate formula.