

MATH 4093/5093 Homework 1 Due Fri, 1/28/11

Problem 1. This problem is mostly an exercise in MATLAB programming. MATLAB is installed on many of the computers on campus, in particular, on the Windows machines in the computer lab in 232 PHSC (which is open from 7:30 a.m. to 12 a.m. on Mon-Thu, 7:30 a.m. to 10 p.m. on Fri, 10 a.m. to 10 p.m. on Sat, and 12 p.m. to 12 a.m. on Sun).

To open MATLAB on a Windows machine, click on ‘All Programs’, then on ‘MATLAB’, then on ‘R2009a’, and finally on ‘MATLAB R2009a’ (or something similar, depending on the version installed on the particular computer). On a Mac, find MATLAB in ‘Applications’ and double-click on it. In Linux, type `matlab` in the command window.

Read pages 2 and 4 of the tutorial *MATLAB Overview* by Ed Overman (available at the class web-site), and try using MATLAB simply as a calculator – for example, try the commands given on page 4 of the tutorial, type

```
format short
pi
format long
pi
format short e
pi
format long e
pi
```

(press RETURN after each line), and look at the output. If you type

```
help format
```

you will see detailed information about this command (the same works for any other MATLAB command). To assign the value $\sqrt{2}$ to the variable `x15`, type

```
x15 = sqrt(2)
```

When you press RETURN, MATLAB will display the output

```
x15 = 1.41421356237310
```

on the screen. If you want to assign the value $\sqrt{2}$ to `x15`, but do not want MATLAB to display the output, type

```
x15 = sqrt(2);
```

From the MATLAB menu, go to ‘File’, and open a new file. Type the following commands in it:

```
function s = sum_odd_numbers(n)
% This code computes the sum of the first n odd integers
    s = 0.0;
    for k = 1:n
        s = s + 2*k - 1
    end;
```

Save the file with the name `sum_odd_numbers.m` in the directory that MATLAB offers you to save it. This file is an example of a MATLAB *function file*. It contains the function `sum_odd_numbers` which expects one argument. All symbols from `%` to the end of the line are comments. Note that the name of the command in the first line of the function file should be the same as the name of the file (except that the file name has the extension `.m`). Call this program from the MATLAB command window by typing `sum_odd_numbers(4)` and the output will be

```
s = 1
s = 4
s = 9
s = 16
ans = 16
```

The program computes the sum of the first `n` positive odd integers, displaying the sum at each step. If you added a semicolon at the end of the line

```
s = s + 2*k - 1
```

then the program will not display the intermediate results (try this!). Since we called our program without assigning its output value to anything, MATLAB assigned the output value to the variable `ans` ('answer'); type `ans` in the command window and press RETURN to see what happens. If you would like to assign the value of the output of the program to, say, the variable `b`, type

```
b = sum_odd_numbers(4);
```

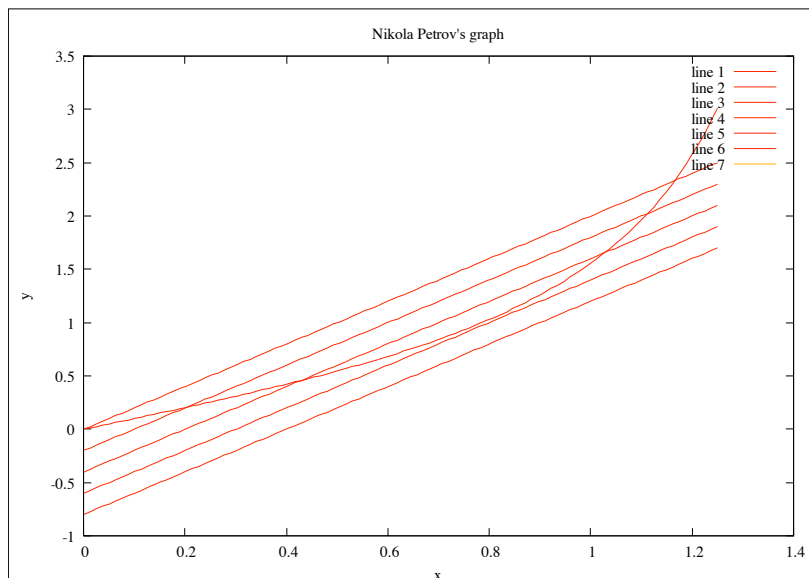
in the command line and press RETURN.

- (a) Suppose that you want to compute e^{-1} by using the Taylor series of the exponential function. How many terms do you need to use if you need accuracy of 10^{-10} ? In other words, find the integer N such that $\left| \sum_{k=0}^N \frac{(-1)^k}{k!} - e^{-1} \right| \leq 10^{-10}$.
- (b) Write a MATLAB file called `inv_exp.m` which computes the value of e^{-1} with accuracy 10^{-10} , displaying the values of all the partial sums $\sum_{k=0}^n \frac{(-1)^k}{k!}$. The input should be the number N you found in part (a). Attach a printout of the file and the output generated by calling it.

Problem 2. Let the functions f and g_α be defined as

$$f(x) = \tan x, \quad g_\alpha(x) = 2x - \alpha. \quad (1)$$

Here α is a parameter (i.e., a constant independent of x) that can take any real value. In the figure below the functions f , g_0 , $g_{0.2}$, $g_{0.4}$, $g_{0.6}$, and $g_{0.8}$ are plotted for values of the argument $x \in [0, 1.25]$. Clearly, for some values of α (namely, for $\alpha = 0, 0.2, 0.4$), the graph



of f intersects the graph of g_α for $x \in [0, 1.25]$; on the other hand, for $\alpha = 0.6, 0.8$, the graphs of f does not intersect the graph of g_α for $x \in [0, 1.25]$. It is clear that for some value of α , say $\alpha = \alpha^*$, the graphs of f and g_{α^*} will have only one common point, and that at this point the graph of g_{α^*} will be tangent to the graph of f . Let x^* stand for the value of the abscissa of the point where the graph of f “touches” the graph of g_{α^*} .

- Derive conditions for the tangency between the two graphs to occur; in other words, derive two equations from which you can find α^* and x^* . Formulate the conditions for general functions f and g_α (not only for the concrete functions given by (1)). Please explain your reasoning.
- Now let f and g_α be the functions given by (1). Solve the equations derived in part (a) to find the values α^* and x^* .
- In MATLAB, plot the graphs of f and g_{α^*} for $x \in [0, 1.25]$. The axis must be labeled by x and y , respectively, and the graph must have a title *Your Name's graph*. All the information that you will need to do this can be found on pages 34–36 of the tutorial *MATLAB Overview* by Ed Overman. Note that since the single quote is a reserved symbol in MATLAB, you have to write `title('Your Name\'s graph')` (note that the single quote is preceded by a backslash, so that MATLAB understands that this is

simply a single quote, not the reserved symbol). First create a vector containing 100 points equally spaced in $[0, 1.25]$ (the first one equal to 0, the last one to 1.25) with the command

```
x = linspace(0,1.25,100);
```

(if you do not put the semicolon at the end of the line, MATLAB will display the values of all the points). Then you can define the array

```
y1 = tan(x);
```

and the array y2 containing the values of g_{α^*} , and plot them both on the same graph following the instructions in the tutorial. Write down the MATLAB commands that you have used to create your graph, and attach a copy of the graph.

Problem 3. Find the limits and the convergence rates as $n \rightarrow \infty$ of the following sequences:

(a) $\lim_{n \rightarrow \infty} \frac{n-1}{n^2+2}$;

(b) $\lim_{n \rightarrow \infty} \frac{n^2}{8n^5-7n}$;

(c) $\lim_{n \rightarrow \infty} \frac{\sin n}{n}$;

(d) $\lim_{n \rightarrow \infty} \left(\sqrt{n+1} - \sqrt{n} \right)$.

Hint: (d) Write $(n+1)^{1/2} - n^{1/2} = n^{1/2} \left[\left(1 + \frac{1}{n}\right)^{1/2} - 1 \right]$, and use that the Taylor expansion of $(1+x)^\alpha$ around $x=0$ when α is not equal to a positive integer is given by

$$(1+x)^\alpha = 1 + \frac{\alpha}{1!}x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \dots .$$