Math 3413.001: Physical Mathematics I

Homework 10, due date TBA

Lecture 21 (Mar 31) : Section 9.3/9.5

1. Find a formal series solution for the following differential equation.

$$x'' + 2x = f(t), \qquad x(0) = 0 = x(2), \qquad f(t) = \begin{cases} 1 & \text{if } 0 < t < 1; \\ 0 & \text{if } 1 \le t < 2. \end{cases}$$

2. Find a formal series solution for the following differential equation.

$$x'' + 2x = f(t), \qquad x'(0) = 0 = x'(2), \qquad f(t) = \begin{cases} 1 & \text{if } 0 < t < 1; \\ 0 & \text{if } 1 \le t < 2. \end{cases}$$

3. The goal is to solve the boundary value problem

$$u_t = 2u_{xx}, \qquad u(0,t) = u(1,t) = 0, \qquad u(x,0) = 31\sin(2\pi x) + 5\sin(3\pi x).$$

   (a) Show that the following functions $u_1(x,t), u_2(x,t), u_3(x,t)$ satisfy $u_t = 2u_{xx}$ and $u(0,t) = u(1,t) = 0$.

$$u_1(x,t) = e^{-2\pi^2 t}\sin(\pi x), \quad u_2(x,t) = e^{-8\pi^2 t}\sin(2\pi x), \quad u_1(x,t) = e^{-18\pi^2 t}\sin(3\pi x).$$

   (b) Find constants c_1, c_2, c_3 such that $u(x,t) = c_1u_1(x,t) + c_2u_2(x,t) + c_3u_3(x,t)$ is a solution to the above boundary value problem.

Suggested problems from the book (DO NOT SUBMIT): Pg 589-590, #12, 14