MATH 4433 Homework 11 Due 11/20/2014

Section 5.3: Exercise 13. Hints and remarks:

- in part (a), suppose that f is continuous and not monotone, and come to a contradiction;
- in part (b), do the proof only in the case when c is not an endpoint of I (if c is an endpoint of I, the proof should be altered slightly – you do not have to do this); you can start like this: since c is not an endpoint of I, from part (a) we know that $f(c)$ is not an endpoint of $f(I)$, therefore there exists $r > 0$ such that $(f(c) - r, f(c) + r) \subseteq I$ (here we are using that I is an interval); given any $\varepsilon > 0$, choose $\alpha := \min\{\varepsilon, \frac{r}{2}\}$, then $(f(c) - \alpha, f(c) + \alpha) \subseteq (f(c) - \varepsilon, f(c) + \varepsilon) \subseteq I$; then let y and z be defined by $f(y) := f(c) - \alpha$ and $z := f(c) + \alpha$; is the open interval (y, z) a subset of I?; proceed to show the existence of $\delta > 0$ such that $|x - c| < \delta$ to imply that $|f(x) - f(c)| < \varepsilon$; sketching the graph of f helps.

Section 6.1: Exercises 8, 13, 14, 15. Hints and remarks:

- in Exercise 8(a) find separately the derivatives $f'(x)$ for $x \neq 0$ and for $x = 0$;
- in Exercises 13 and 14 you may use any of the results proved in Section 6.1, but you have to write specifically which results you have used.

Section 6.2: Exercises 4, 5(a,b,c), 6, 9, 11, 13, 20. Hints and remarks:

- in Exercise 6, just draw clearly the graphs of three functions each of which violates exactly one of the three conditions required in Rolle's Theorem and state which condition each function violates; no further explanation is required;
- in Exercise 9 you simply have to write down two specific counterexamples to show that from the fact that a function is strictly increasing (resp. decreasing) on an interval I, one cannot conclude that the derivative of the function is positive (resp. negative) everywhere on I;
- to prove the claim in Exercise 11, assume that f is not strictly increasing, which would imply that there exist points $x_1 < x_2$ in $[a, b]$ such that $f(x_1) = f(x_2)$; this fact together with the assumption that f' is nonnegative will quickly lead to a contradiction;
- Exercise 20 is a simple calculation that can be found in any elementary Calculus textbook.

Food for Thought:

- Sec. **6.2**, exercises 1, 2.