MATH 4443 **Homework 11** **Not to be turned in**

All examples and problems below are from the book [SV] of Satish Shirali and Harkrishan Lal Vasudeva, *Multivariable Analysis*, Springer, 2011.

Problem 1. Read Example 3-2.6 ([SV], page 82), in which you will prove that a function is not differentiable by showing that its directional derivative $D_{\mathbf{b}}\vec{f}(\mathbf{x})$ is not linear in $\mathbf{b}$ (see Remark 3-2.5(c) of [SV]).

Problem 2. Problem 3-2.P3 ([SV], pages 84, 316) in which you will see an example of a function that has directional derivatives in all directions but is not continuous (and, therefore, not differentiable).

Problem 3. Problem 3-2.P4 ([SV], pages 84, 316) in which you will see an example of a function that is continuous but not differentiable.

Problem 4. Problem 3-2.P12 ([SV], pages 85, 319) in which you will see a very simple example of a function that has partial derivatives but is not differentiable (and not even continuous). If you imagine the graph of the function, the fact that it is not continuous will become perfectly clear.

Problem 5. Read Example 3-3.2 ([SV], pages 86-88) for simple applications of the Chain Rule.

Problem 6. Proving the differentiability of a function directly from the definition of derivative is a difficult and tedious task. It is much simpler to use Theorem 3-4.4 stated on page 99 of [SV] which says that if all first partial derivatives of a function exist and are continuous, then the function is differentiable. Read Example 3-4.6(a) on page 102 of [SV], which uses this theorem.

Problem 7. Problem 3-4.P11 ([SV], pages 105-106, 326-327) in which you will compute the linearization of a function of two variables.

Problem 8.

(a) Problem 3-4.P17 ([SV], pages 107, 328) in which you will derive the so-called *Leibniz's formula* which allows you to differentiate a definite integral whose integrand depends on a parameter, with respect to the parameter.

(b) Problem 3-4.P19 ([SV], pages 107, 329) in which you will use Leibniz's formula in a concrete example (and will derive an integral representation of the so-called *Bessel functions* which are very useful in the solution of many partial differential equations).

(c) Problem 3-4.P20 ([SV], pages 107, 329) in which you will generalize Leibniz's formula to the case when the limits of integration are also functions of the parameter.