Additional Problem.

Let \mathcal{L} stand for the set of all linear functions of one variable, i.e., \mathcal{L} consists of all functions of the form f(x) = ax + b (where a and b are constants). Clearly, the quadratic function $q(x) = x^2$ does not belong to \mathcal{L} . Suppose that you would like to find a linear function f from \mathcal{L} that is the best approximation of q on the interval [0,1], in the sense that f is such that the "error" defined by the integral

$$\int_0^1 [q(x) - f(x)]^2 dx = \int_0^1 [q(x) - (ax + b)]^2 dx = \frac{a^2}{3} + b^2 + ab - \frac{a}{2} - \frac{2b}{3} + \frac{1}{5}.$$

- (a) Find the values of a and b for which the "error" is the smallest. Hint: The answer is a = 1, $b = -\frac{1}{6}$, but you have to show me how you can obtain these values.
- (b) Based on your answer in part (a) (which was actually given in the hint), write down the function f from \mathcal{L} that is "closest" to the function q, where by "distance" between f and q we mean the "error" defined in part (a).
- (c) What is the value of the "error" in approximating the function q(x) with the function obtained in part (b)?