MATH 2513 **Homework 4** **Due 2/14/19 (Thursday)**

Please explain briefly but clearly your reasoning (unless it is totally obvious from your answer, i.e., when you have to list the elements of a set or to draw a set in the plane).

Please write the problems in the same order as they are given in the assignment.

Note that the odd-numbered problems have answers at the end of Hammack's book. I strongly suggest that you do *all* odd-numbered problems for practice; moreover, many of them are very similar to the assigned homework problems.

Hammack, Section 3.2: Exercises 4, 6, 8, 10.

Remark: In Exercise 6 you are asked four different questions. Please denote them by (a)–(d) and answer them separately. Briefly (but clearly) explain your reasoning.

Additional problem 1. Use the logical equivalences from page 52 of Hammack's book to simplify the following statements. In each step indicate which of the laws you have used. Use **T** to denote Tautology and **C** to denote Contradiction.

(a) $(P \wedge \sim Q) \vee [(\sim P) \wedge \sim Q]$

(b) $(P \vee \sim Q) \wedge [(\sim P) \vee \sim Q]$

(c) $\{P \wedge [\sim ((\sim P) \vee Q)]\} \vee (P \wedge Q)$

Additional problem 2.

(a) Write down the tautology on which Modus tollens is based.

(b) Fill the table shown below; decide what you have to write in the rightmost column. Discuss.

P	Q	$P \Rightarrow Q$	$\sim Q$	$(P \Rightarrow Q) \wedge (\sim Q)$	

Additional problem 3. Use truth tables to see if the following forms of argument are valid.

(a)

$$\begin{array}{l} P \Rightarrow Q \\ Q \\ \hline \therefore P \end{array}$$

(b)

$$\begin{array}{l} P \Rightarrow Q \\ \underline{\sim P} \\ \therefore \sim Q \end{array}$$

Additional problem 4.

Definition. A sequence $(a_n)_{n=1}^{\infty}$ is said to converge to a if

$$\forall \varepsilon > 0,\ \exists K,\ \forall n > K,\ |a_n - a| < \varepsilon.$$

(a) Prove that the sequence $(a_n)_{n=1}^{\infty}$ defined by $a_n = \frac{3}{n^5}$ converges to 0. Please be specific (in particular, you have to choose a specific value of K, and show that it works).

(b) Write in mathematical notation what it means that a sequence $(a_n)_{n=1}^{\infty}$ does *not* converge to a.

(c) Prove that the sequence from part (a) does not converges to 10^{-100}. Again, be specific!