MATH 4433 Homework 4 Due 9/23/2014

Please read carefully:

- **This is the *final* version of this homework (updated on Thursday, Sep 18)!**
- **The problems marked with ★ do *not* have to be turned in (on Tuesday, Sep 16, I distributed in class solutions of these problems).**

Section 2.3: Exercises 8, 10, 11★, 12, 13★, 17, 19★, 21(a,b), 22★, 32. Hints and remarks:

- the inclusion in Exercise 21(a) does not hold, so you have to construct a counterexample; the inclusion in Exercise 21(b) is valid, so you have to prove it;
- recall that "iff" means "if and only if", so in part (a) you have to prove two things: (1) if f has a left inverse, then f is injective, and (2) if f is injective, then f has a left inverse; similarly for part (b).

Section 2.4: Exercises 3(e,f), 4★, 5, 9, 10. Hints and remarks:

- in Exercise 3(e) you can use that if $A = (0, \frac{\pi}{2})$, $B = (0, \infty)$, the function $\tan : A \to B$ is a bijection; in Exercise 3(f) use the result of Exercise 4 and the fact that tan maps $(-\frac{\pi}{2}, \frac{\pi}{2})$ bijectively onto $\mathbb{R}$;
- for Exercise 9(a), each polynomial $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ of degree n with integer coefficients is completely defined by its coefficients,

  $$(a_0, a_1, \ldots, a_{n-1}, a_n) \in \underbrace{\mathbb{Z} \times \mathbb{Z} \times \cdots \times \mathbb{Z}}_{n+1 \text{ times}} .$$

  Use this and the fact that a countable family of countable sets is countable to come to the desired conclusion; please be *completely specific* in your explanation!
- in Exercise 10, recall our discussion in class of the strange properties of infinite sets.

Section 3.1: Exercises 23(b), 24, 30. Hints and remarks:

- the inequality in Exercise 23(b) is true for all $n \in \mathbb{N} \setminus \{3\}$; verify it for $n = 1$ and $n = 2$ separately ("by hand"); apply induction (Theorem 3.1.6) for $n \geq 4$;
- in Exercise 30(a) note that, by definition, $0! = 1$, so that $\binom{n}{n} = \binom{n}{0} = 1$ for all $n \in \mathbb{N}$.

Section 3.2: Exercise 3(b,c,g,h). Hints and remarks:

- note that in each of parts (b) and (c) you have to prove two things;
- when you are solving some part of this problem, you are allowed to use Axioms A1–A5, M1–M5, DL, O1–O4, all parts of Theorem 3.2.2, and all parts of Exercise 3 that precede the part you are proving at the moment, including the parts of the problem that have a star (i.e., have hints or solutions in the back of the book); you do *not* need to write solutions of the starred ones.

Food for Thought:

- Sec. **2.3**, exercises 26, 27, 28.
- Sec. **2.4**, exercises 2, 5, 11.
- Sec. **3.1**, exercises 13, 23(a).
- Sec. **3.2**, exercises 3(a,e,f,l).