MATH 5463 Homework 4 Due Thu, Feb 26

Problems 6, 8, 12 from Section 6.1 of the book.

Remark to Problem 8: In the statement in this problems it is mentioned that if $\mu(X) = 1$ and $f \in L^p$, then $f \in L^q$ for all $q \in (0, p)$. This is a direct consequence of Proposition 6.12, so that you can take it for granted.

Remark to Problem 12: For the Parallelogram Law see page 173 of Folland's book.

Additional problem 1. Let a and b be real numbers with $a < b$, and $C([a, b], \mathbb{R})$ stands for the set of real-valued continuous functions on$[a, b]$.

(a) Show by example that the set $C([a, b], \mathbb{R})$ equipped with the L^1-norm is not a Banach space.

(b) Prove that $C([a, b], \mathbb{R})$ endowed with the L^∞-norm is a Banach space. (This follows from Theorem 6.8 of the book, but you have to give a direct proof, using some results about continuous functions.)

Additional problem 2. Let $\|\cdot\|_p$ stand for the ℓ^p norm on $\mathbb{C}^n$.

(a) Show that the ℓ^1 and ℓ^∞ norms on $\mathbb{C}^n$ are equivalent.

(b) Show that the ℓ^2 and ℓ^∞ norms on $\mathbb{C}^n$ are equivalent.

(c) Use (a) and (b) to show that the ℓ^1 and the ℓ^2 norms on $\mathbb{C}^n$ are equivalent.

Food for thought. Read the definitions on pages 113 and 114 of Folland's book related to topological spaces, the definition of Hausdorff space on page 117, as well as the text on page 128 about compact spaces (before looking at the proofs of Propositions 4.21–4.24, try to do them yourself).