Homework 5 – Additional problems

Problem 1. Use Mathematical Induction to prove the useful identity

$$\sum_{j=1}^{n-1} q^j = \frac{q^n - 1}{q - 1} \qquad \text{for } q \neq 1\,,\ n \in \mathbb{N}\,.$$

Problem 2. Let $\prod_{j=1}^{n} a_j$ stand for the product $a_1 a_2 \cdots a_n$. Use Mathematical Induction to prove that

$$\prod_{j=2}^{n} \left(1 - \frac{1}{j^2}\right) = \frac{n+1}{2n} \qquad \text{for } n \in \{2, 3, 4, \cdots\}\,.$$

Problem 3. Let a_1, a_2, a_3, ... be a sequence of numbers defined recursively by

$$a_1 = 1\,, \qquad a_{n+1} = a_n + 3n(n+1) \quad \text{for } n \in \mathbb{N}\,.$$

Prove that

$$a_n = n^3 - n + 1 \quad \text{for } n \in \mathbb{N}\,.$$