MATH 4433 Homework 5 Due 10/2/2014

Section 3.3: Exercises 4, 6(a), 8, 10, 13. Hints and remarks:

- in Exercise 4, justify briefly (but convincingly) your answers only in parts (a), (c), (e), and (l); in all other parts simply write down the infimum and the minimum (or state that the set has no infimum and/or minimum);
- to prove the uniqueness of the supremum of the nonempty set $S \subseteq \mathbb{R}$ in Exercise 6, assume that there are two distinct suprema of S; in other words, assume that there exist two numbers m and $\widetilde{m}$ such that $m \neq \widetilde{m}$ and that each of them is supremum of S (i.e., satisfies the two conditions in Definition 3.3.5), and show that this leads to a contradiction; you may use the Trichotomy Law (Axiom O1) and show that assuming that $m < \widetilde{m}$ or that $\widetilde{m} < m$ leads to a contradiction;
- in Exercise 8, prove only that $\inf T \leq \inf S$ and that $\inf S \leq \sup S$; do *not* do the proof that $\sup S \leq \sup T$ (which is completely analogous to the proof that $\inf T \leq \inf S$);
- in Problem 10(b), use that the irrational numbers are dense in $\mathbb{R}$ (Theorem 3.3.15); there is no need to prove Theorem 3.3.15 in your homework.

Section 3.4: Exercises 5(a,b,c), 12, 19, 21, 23(a,b,c,e). Hints and remarks:

- in Exercise 5(a,b,c), find the boundary of the set and use Definition 3.4.6 to determine if the set is open or closed;
- the fact in Exercise 19 is true – prove it by using the definition of supremum;
- Exercise 21 follows easily from the density of $\mathbb{Q}$ in $\mathbb{R}$;
- in Exercise 23:
  - give a detailed proof of part (a);
  - part (b) follows from part (a) and Theorem 3.4.7 – explain why;
  - in part (c) you have to prove that $\operatorname{int}(S \cap T) \subseteq (\operatorname{int} S) \cap (\operatorname{int} T)$ and that $(\operatorname{int} S) \cap (\operatorname{int} T) \subseteq \operatorname{int}(S \cap T)$;
  - do *not* prove part (d)!;
  - give an explicit example for part (e).

Please turn the page!

Food for Thought:

- Sec. **3.3**, exercises 3, 6(b).
- Sec. **3.4**, exercises 3, 4, 5(d,e,f), 8 (one needs to show that [a $\Rightarrow$ ($\sim$b) $\wedge$ ($\sim$c)], [b $\Rightarrow$ ($\sim$c) $\wedge$ ($\sim$a)], [c $\Rightarrow$ ($\sim$a) $\wedge$ ($\sim$b)]).
- Sec. **3.4**: the proof of Theorem 3.4.7 in the book is extremely short (four lines) – try to prove the claims of the theorem without looking at the proof in the book; the fact in Exercise 3.4/8 is very useful to have in mind when you are working on such proofs.