

General Remark. Whenever you have to use the definition of convergence of a sequence (Definition 2.2.3 in Abbot), use the simplified version (slightly different from the one in the book)

$$(a = \lim a_n) \Leftrightarrow (\forall \varepsilon > 0 \exists N \text{ s.t. } |a_n - a| < \varepsilon \forall n > N)$$

Abbott, Section 2.2:

Exercise 2.2.2 (page 47).

Hint: (b) $\frac{2n^2}{n^3 + 3} < \frac{2n^2}{n^3}$; (c) $|\sin(n^2)| \leq 1$.

Abbott, Section 2.3:

Exercises 2.3.1 (see the Remark below), 2.3.2, 2.3.4, 2.3.7, 2.3.8, 2.3.10(b,d), 2.3.12(a,c) (pages 54–56).

Remarks and hints:

- Exercise 2.3.1: in part (a) you have to use Definition 2.2.3 directly; in part (b), notice that $x_n = (\sqrt{x_n})^2$ and use Theorem 2.3.3.
- Exercise 2.3.4(b): Use some elementary algebra before applying the Algebraic Limit Theorem.

Additional Problem.

Let $f : A \rightarrow B$ be a given function. Define a relation F on A by

$$x F y \quad \text{iff} \quad f(x) = f(y) .$$

- Prove that F is an equivalence relation on A .
- For any $x \in A$, let E_x be the equivalence class of x : $E_x = \{y \in A : y F x\}$.
Let E be the collection of all equivalence classes, i.e., $E = \{E_x : x \in A\}$.
Prove that the function $g : A \rightarrow E$ defined by $g(x) = E_x$ is surjective.
- Prove that the function $h : E \rightarrow B$ defined by $h(E_x) = f(x)$ is injective.
- Prove that $f = h \circ g$, i.e., that $f(x) = h(g(x))$ for all $x \in A$. Thus, we conclude that each function can be written as the composition of a surjective function and an injective function.
- Let A be the set of all students at OU. Define the function $f : A \rightarrow \{0, 1, 2, 3, \dots, 200\}$ by “ $f(x)$ is the age of x ” (we assume that the age is measured in integer number of years). Describe the functions h and g as given above.