MATH 2513 Homework 6 Due 3/7/19 (Thursday)

Please explain briefly but clearly your reasoning (unless it is totally obvious from your answer, i.e., when you have to list the elements of a set or to draw a set in the plane).

Please write the problems in the same order as they are given in the assignment.

Note that the odd-numbered problems have answers at the end of Hammack's book. I strongly suggest that you do *all* odd-numbered problems for practice; moreover, many of them are very similar to the assigned homework problems.

Hammack, Section 3.6:

- Exercise 6 – give only a *combinatorial proof*, interpreting the numbers as cardinalities of some subsets,
- Exercise 10,
- Exercise 12 – solve it in two ways: (a) by using Fact 3.5 on page 85 of the book, and (b) by interpreting the binomial coefficients as the number of subsets of a given set.

Hammack, Section 3.7:

- Exercise 2,
- Exercise 4(a),
- Exercise 5 (explain all the numbers in the solution in the back of the book),
- Exercise 8.

Hammack, Chapter 4: Exercises 4, 8 (hint: see the solution of Exercise 9).

Additional Problem 1. Prove that there are two people living in Oklahoma who have the same birthday and birthmonth and birthyear.

Additional Problem 2. Show that for every natural number $n \in \mathbb{N}$ there is a multiple of n that has only 0s and 1s in its decimal expansion.
Follow the following steps:

(a) If we divide a natural number m by n, what values can the remainder take? (See the Division Algorithm on page 117 of the book.)

(b) Prove that if the remainders of the division of m and k by n are equal, then $(m-k)$ is divisible by n.

(c) Consider the $(n+1)$ integers 1, 11, 111, 1111, ..., $11\cdots 1$, where the last integer in this list is the integer with $(n+1)$ 1s in its decimal expansion. If we divide each one by n, what can you say about the remainders?

(d) Use what you showed in parts (a), (b), and (c) to prove that among the $(n+1)$ numbers considered in part (c) there are two numbers whose difference is divisible by n. What are the digits of this difference?

(e) Use the method developed in parts (a)–(c) to construct a number written with only 0s and 1s that is a multiple of 7.

Additional Problem 3. Prove that if $a \mid b$ and $a \mid (b^2 + c)$, then $a \mid c$.