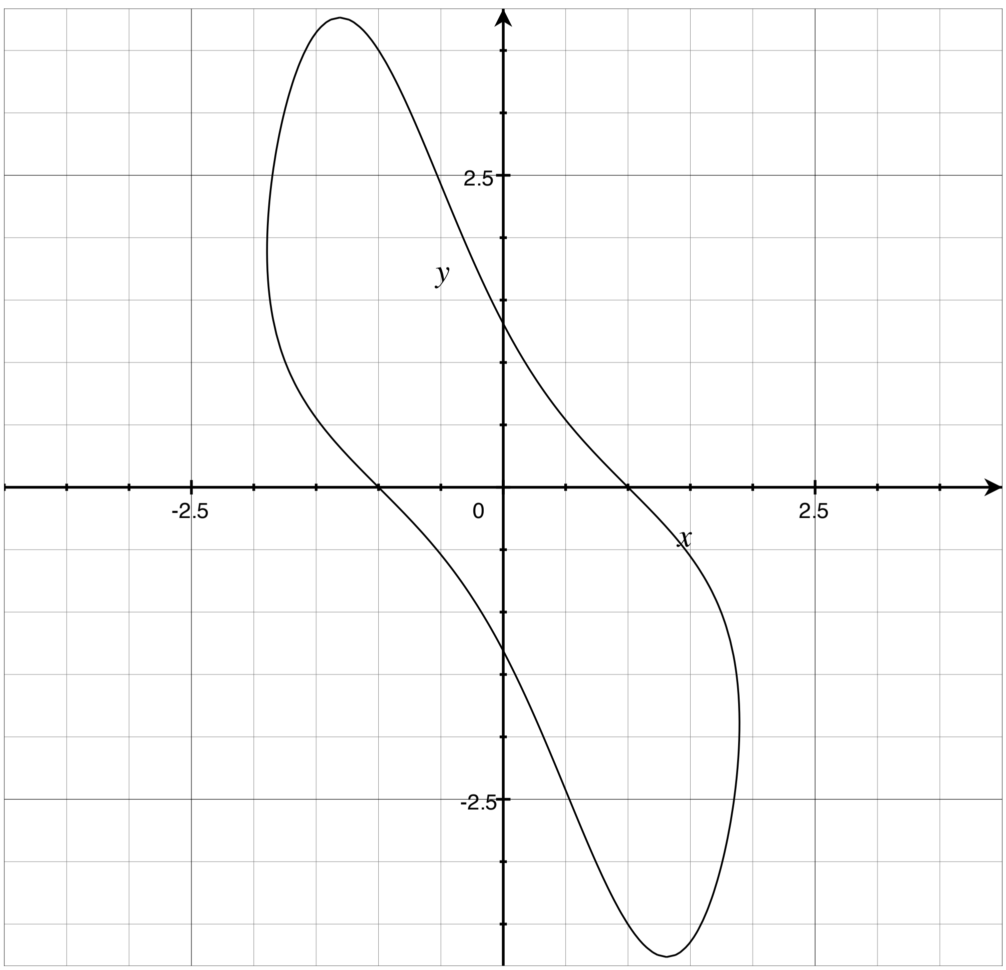
MATH 3413 Homework 6 Due Thu, Oct 12

Sec. 7.1: problems 3, 8, 12, 14, 19, 20, 25, 28, 38 (after you find the solution of this problem for general a and b, find it for $a = 1$, $b = 2$ and compare with your result in problem 7.1/8).

Sec. 7.2: problems 2, 5, 17, 19 (solve problems 17 and 19 both by applying Theorem 2 and by by using the method of partial fractions).

Additional problem 1. The function $y(x)$ whose graph is plotted in the figure below is given implicitly by the equation

$$x^2 + 2x \arctan y + \ln(1 + y^2) = 1 \ .$$



(a) Find a first-order ODE that the function $y(x)$ satisfies.

(b) Find the numerical values of $y(0)$ (as you see from the figure, there are two such values).

Additional problem 2.

(a) Find the general solution $\mathbf{X}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ of the first order system

$$\mathbf{X}'(t) = \begin{pmatrix} 0 & 1 \\ 6 & -1 \end{pmatrix} \cdot \mathbf{X}(t)$$

by first converting it to a second order differential equation.

(b) Find the solution of the initial value problem

$$\mathbf{X}'(t) = \begin{pmatrix} 0 & 1 \\ 6 & -1 \end{pmatrix} \cdot \mathbf{X}(t) \,, \qquad \mathbf{X}(0) = \mathbf{X}^{(0)} = \begin{pmatrix} x_1^{(0)} \\ x_2^{(0)} \end{pmatrix} \,. \tag{1}$$

(c) For most initial conditions $\mathbf{X}^{(0)}$ the solution $\mathbf{X}(t)$ of the initial value problem (1) will go to infinity. However, there is a line in the (x_1, x_2)-plane such that, $\mathbf{X}^{(0)}$ belongs to this line, $\mathbf{X}(t)$ will stay bounded for all t. Write down the equation of this line.