Homework 7 – Additional problems, part 1

Problem 1. The *complete elliptic integral of the second kind* is the function

$$E(k) = \int_0^{\pi/2} \sqrt{1 - k \sin^2 \theta}\, \mathrm{d}\theta \,, \qquad k \in [-1, 1] \,.$$

(a) Assume that the values of $E(k)$ are known, and express the value of

$$\int_0^{\pi/2} \sqrt{1 - \tfrac{1}{4} \cos^2 \theta}\, \mathrm{d}\theta$$

in terms of $E(k)$.

(b) Rewrite the definition of $E(k)$ by making the substitution $t = \sin \theta$.