Math 3413.001: Physical Mathematics I

Homework 7, due March 12 (Thursday)

Lecture 15 (Mar 3) Due date 03/12/2020 : Section 7.5

1. Find the inverse Laplace transform \( f(t) \) of \( F(s) = \frac{e^{-2s}}{s-3} \), and sketch the graph of \( f(t) \).
2. Let 
\[
f(t) = \begin{cases} 
0 & \text{if } 0 \leq t < 3; \\
2 & \text{if } 3 \leq t < 5; \\
0 & \text{if } 5 \leq t.
\end{cases}
\]
Sketch the graph of \( f(t) \), rewrite \( f(t) \) in terms of the step functions \( u_a(t) \), and then find the Laplace transform of \( f(t) \).
3. Let \( f(t) \) be the periodic function with period 2 with \( f(t) = 1 \) for \( 0 \leq t < 1 \) and \( f(t) = 0 \) for \( 1 \leq t < 2 \). Sketch the graph of \( f(t) \). Find the Laplace transform of \( f(t) \).
4. Let \( f(t) \) be the periodic function with period 2 with \( f(t) = t \) for \( 0 \leq t < 1 \) and \( f(t) = 0 \) for \( 1 \leq t < 2 \). Sketch the graph of \( f(t) \). Find the Laplace transform of \( f(t) \).

Suggested problems from the book (DO NOT SUBMIT): Pg 482-483, #3, 8, 12, 16, 26, 28, 31

Lecture 16 (Mar 5) Due date 03/12/2020 : Section 7.6

1. Solve the initial value problem
\[
x'' + x' - 2x = \delta(t) + \delta(t - 2) \quad x(0) = x'(0) = 0.
\]
2. Solve the initial value problem
\[
x'' + 6x' + 9x = \delta(t) \quad x(0) = x'(0) = 0.
\]
3. Apply Duhamel’s principle to write an integral formula for the solution of the initial value problem
\[
x'' + 4x' + 13x = f(t) \quad x(0) = x'(0) = 0.
\]
4. Find a non-trivial solution to
\[
tx'' + (t - 2)x' + x = 0
\]
such that \( x(0) = 0 \).

\textit{Hint:} See Example 5 in Section 7.4.

Suggested problems from the book (DO NOT SUBMIT): Pg 492-493, #2, 6, 8, 19b