Math 3413.001: Physical Mathematics I

Homework 7, due March 12 (Thursday)

Lecture 15 (Mar 3) Due date 03/12/2020 : Section 7.5

1. Find the inverse Laplace transform $f(t)$ of $F(s) = \frac{e^{-2s}}{s-3}$, and sketch the graph of $f(t)$.
2. Let
$$f(t) = \begin{cases} 0 & \text{if } 0 \le t < 3; \\ 2 & \text{if } 3 \le t < 5; \\ 0 & \text{if } 5 \le t. \end{cases}$$
Sketch the graph of $f(t)$, rewrite $f(t)$ in terms of the step functions $u_a(t)$, and then find the Laplace transform of $f(t)$.
3. Let $f(t)$ be the periodic function with period 2 with $f(t) = 1$ for $0 \le t < 1$ and $f(t) = 0$ for $1 \le t < 2$. Sketch the graph of $f(t)$. Find the Laplace transform of $f(t)$.
4. Let $f(t)$ be the periodic function with period 2 with $f(t) = t$ for $0 \le t < 1$ and $f(t) = 0$ for $1 \le t < 2$. Sketch the graph of $f(t)$. Find the Laplace transform of $f(t)$.

Suggested problems from the book (DO NOT SUBMIT): Pg 482-483, #3, 8, 12, 16, 26, 28, 31

Lecture 16 (Mar 5) Due date 03/12/2020 : Section 7.6

1. Solve the initial value problem
$$x'' + x' - 2x = \delta(t) + \delta(t-2) \qquad x(0) = x'(0) = 0.$$
2. Solve the initial value problem
$$x'' + 6x' + 9x = \delta(t) \qquad x(0) = x'(0) = 0.$$
3. Apply Duhamel's principle to write an integral formula for the solution of the initial value problem
$$x'' + 4x' + 13x = f(t) \qquad x(0) = x'(0) = 0.$$
4. Find a non-trivial solution to
$$tx'' + (t-2)x' + x = 0$$
such that $x(0) = 0$.

Hint: See Example 5 in Section 7.4.

Suggested problems from the book (DO NOT SUBMIT): Pg 492-493, #2, 6, 8, 19*b*