MATH 4433 **Homework 7** **Due 3/23/17 (Thursday)**

Abbott, Section 2.4:

Exercise 2.4.4(b) (page 60).

Abbott, Section 2.5:

Exercise 2.5.4 (page 65).

Abbott, Section 2.6:

Exercises 2.6.2(a,b,d), 2.6.3, 2.6.5 (page 70).

Remarks and hints:

- Exercise 2.6.3: (a) follow the method of proof of Theorem 2.3.3(ii); (b) use the fact that a Cauchy sequence is bounded and the method of proof of Theorem 2.3.3(iii).
- Exercise 2.6.5: (a) use what you know from Calculus about the harmonic series.

Abbott, Section 3.2:

Exercises 3.2.2, 3.2.3, 3.2.4(a), 3.2.6(a,c,d), 3.2.10 (pages 93–95).

Remarks and hints:

- Exercise 3.2.2: justify briefly your answers.
- Exercise 3.2.3: use the results of Examples 2.4.4 and 2.4.5 on pages 57, 58 of Abbott.

In the additional problems below, you will need the following

Definition. A point x is a *boundary point* of the set A if for all $\varepsilon > 0$, $V_\varepsilon(x) \cap A \neq \varnothing$ and $V_\varepsilon(x) \cap A^c \neq \varnothing$. The set of all boundary points of A is denoted by ∂A.

Examples: $\partial\,[0,1) = \{0,1\}$, $\partial\,\{\frac{1}{n} : n \in \mathbb{N}\} = \{0\} \cup \{\frac{1}{n} : n \in \mathbb{N}\}$, $\partial\,(\mathbb{Q} \cap [0,1)) = [0,1]$.

Additional Problem 1.

Let A and B be subsets of $\mathbb{R}$.

(a) Prove that $(x \in \overline{A}) \Leftrightarrow (\forall \varepsilon > 0\,,\ V_\varepsilon(x) \cap A \neq \varnothing)$.

(b) Use the criterion established in part (a) to prove that $\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$.

(c) Find an example to show that equality need not hold in part (b).

(d) Use the criterion established in part (a) to prove that $\partial A = \overline{A} \cap \overline{A^c}$.

Additional Problem 2.

In all parts of this problem, A and B are subsets of $\mathbb{R}$, $\overline{A}$ stands for the closure of A. Find a concrete counterexample for each of the following. Explain briefly.

(a) If A consists of isolated points only, then A is closed.

(b) Every open set contains at least two points.

(c) $\partial\,\overline{A} = \partial A$

(d) $\partial(\partial A) = \partial A$

(e) $\partial\,(A \cup B) = (\partial A) \cup (\partial B)$

(f) $\partial\,(A \cap B) = (\partial A) \cap (\partial B)$