MATH 4433 Homework 7a NOT Due 3/28/17 (Tuesday)

Abbott, Section 3.3:

Exercise 3.3.8 (page 101).

Additional Problem 1.

According to the Nested Compact Set Property, the intersection of any nested sequence of nonempty compact sets is nonempty. In this problem you will construct examples that demonstrate that compactness is indeed necessary in this property.

(a) Find a family of nested closed intervals $\{A_n : n \in \mathbb{N}\}$ such that $\displaystyle\bigcap_{n\in\mathbb{N}} A_n = \varnothing$.

(b) Find a family of nested bounded intervals $\{A_n : n \in \mathbb{N}\}$ such that $\displaystyle\bigcap_{n\in\mathbb{N}} A_n = \varnothing$.

Additional Problem 2.

If K is a compact subset of $\mathbb{R}$ and C is a closed subset of K, then C is compact.

(a) Prove this by using the definition of compactness.

(b) Prove this by using the characterization of compactness from Theorem 3.3.4.

(c) Prove this by using the characterization of compactness that a set is compact if and only if every open cover has a finite subcover (recall the Heine-Borel Theorem).

Additional Problem 3.

Find an uncountable open cover $\{O_\lambda : \lambda \in \Lambda\}$ of $\mathbb{R}$ which has no finite subcover. Does this set have a countable subcover?

Additional Problem 4.

True of false? If true, justify; if false, give a counterexample.

(a) Every finite set is compact.

(b) No infinite set is compact.

(c) If K is compact and x is an accumulation point of K, then $x \in K$.

(d) If A is unbounded, then it has at least one limit point.

(e) If a set K is compact, then $\overline{K} = K$.