Homework 8 – Additional problems, part 1

Problem 1. In this problem you will use quadratic polynomial interpolation to compute approximately the value of the integral

$$I_{\text{exact}} = \int_0^{\pi/2} \cos x \, \mathrm{d}x$$

(of course, you know the value of this integral, so the purpose of this problem is purely illustrative).

(a) Find the unique quadratic polynomial

$$p(x) = Ax^2 + Bx + C$$

which passes through the points $(0, \cos 0)$, $\left(\frac{\pi}{3}, \cos\frac{\pi}{3}\right)$, and $\left(\frac{\pi}{2}, \cos\frac{\pi}{2}\right)$, i.e., through

$$(0, 1) \ , \quad \left(\tfrac{\pi}{3}, \tfrac{1}{2}\right) \ , \quad \left(\tfrac{\pi}{2}, 1\right) \ .$$

Remark: Since the calculation is relatively long and tedious, after you obtain the values of A, B, and C, it would be a good idea to check directly that the polynomial $p(x)$ satisfies the three conditions.

(b) Use the polynomial $p(x)$ to compute the value of the integral

$$I_{\text{approx}} = \int_0^{\pi/2} p(x) \, \mathrm{d}x \ .$$

(c) Compute the relative error of your approximation,

$$\frac{|I_{\text{approx}} - I_{\text{exact}}|}{|I_{\text{exact}}|} \ .$$

Remark: The functions $\cos x$ and $p(x)$ are plotted in the figure below; their values coincide at $x = 0$, $x = \frac{\pi}{3}$, and $x = \frac{\pi}{2}$.