MATH 3413 Homework 8 Due Mon, Nov 8, 2010

Sec. 8.1: problem 4.

Sec. 9.1: problems 2, 9, 10, 11, 13, 27.

Additional problem 1.

Let $\mathbb{R}^2$ stands for the vector space of all two-dimensional vectors, $\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$. Let the inner product in $\mathbb{R}^2$ be given by

$$(\mathbf{u}, \mathbf{v}) := \sum_{i=1}^{2} \sum_{j=1}^{2} u_i \, a_{ij} \, v_j \ ,$$

where

$$a_{11} = 2 \ , \quad a_{12} = a_{21} = 1 \ , \quad a_{22} = 4 \ .$$

Let $\{\mathbf{v}^{(1)}, \mathbf{v}^{(2)}\}$ be a basis in $\mathbb{R}^2$, where $\mathbf{v}^{(1)} = \begin{pmatrix} v_1^{(1)} \\ v_2^{(1)} \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, and $\mathbf{v}^{(2)} = \begin{pmatrix} v_1^{(2)} \\ v_2^{(2)} \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

(a) Check that $\{\mathbf{v}^{(1)}, \mathbf{v}^{(2)}\}$ is an orthogonal basis, i.e., that the inner product of $\mathbf{v}^{(1)}$ and $\mathbf{v}^{(2)}$ is zero: $(\mathbf{v}^{(1)}, \mathbf{v}^{(2)}) = 0$.

(b) Find $(\mathbf{v}^{(1)}, \mathbf{v}^{(1)})$ and $(\mathbf{v}^{(2)}, \mathbf{v}^{(2)})$.

(c) If $\mathbf{u} = \begin{pmatrix} 9 \\ -1 \end{pmatrix} = \alpha_1 \mathbf{v}^{(1)} + \alpha_2 \mathbf{v}^{(2)}$, then find α_1 and α_2 by solving the system of linear equations for them coming from

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \alpha_1 \begin{pmatrix} v_1^{(1)} \\ v_2^{(1)} \end{pmatrix} + \alpha_2 \begin{pmatrix} v_1^{(2)} \\ v_2^{(2)} \end{pmatrix} \ .$$

(d) Independently of part (c), if $\mathbf{u} = \begin{pmatrix} 9 \\ -1 \end{pmatrix} = \alpha_1 \mathbf{v}^{(1)} + \alpha_2 \mathbf{v}^{(2)}$, find α_1 and α_2 by using the formula

$$\alpha_j = \frac{\left(\mathbf{u}, \mathbf{v}^{(j)}\right)}{\left(\mathbf{v}^{(j)}, \mathbf{v}^{(j)}\right)} \ .$$