Math 3413.001: Physical Mathematics I

Homework 8, due March 26 (Thursday)

Lecture 17 (Mar 10) Due date 03/26/2020 : Section 7.6

1. Solve the initial value problem

$$x'' + 3x' + 2x = 7u_{11}(t) - 5\delta(t-2), \qquad x(0) = 2, \quad x'(0) = 1.$$

2. This problem considers a mass m, initially at rest at the origin, that receives an impulse p at time $t = 0$.

   (a) Find the solution $x_\epsilon(t)$ of the initial value problem

$$mx''(t) = pd_{0,\epsilon}(t), \qquad x(0) = 0, \quad x'(0) = 0,$$

   where the function $d_{0,\epsilon}(t)$ is defined on page 485 of the book.

   (b) Show that $\lim_{\epsilon \to 0+} x_\epsilon(t)$ agrees with the solution of the problem

$$mx''(t) = p\delta(t), \qquad x(0) = 0, \quad x'(0) = 0.$$

   (c) If $x(t)$ is the solution of the IVP from part (b), show that $mx'(t) = p$.

3. In this problem you will give another proof of the fact that, for any $a > 0$, $u_a' = \delta_a$. Solve the IVP

$$x' = \delta_a(t), \qquad x(0) = 0$$

   by using Laplace transform, and interpret your result.

4. This problem deals with a mass m on a spring (with elastic constant k) that receives an impulse $p_0 = mv_0$ at time $t = 0$; the mass is initially at rest. Show that the IVPs

$$mx'' + kx = 0, \qquad x(0) = 0, \quad x'(0) = v_0$$

   and

$$mx'' + kx = p_0\delta(t), \qquad x(0) = 0, \quad x'(0) = 0$$

   have the same solution. Thus the effect of the term $p_0\delta(t)$ is, indeed, to impart to the particle an initial moment p_0.

Suggested problems from the book (DO NOT SUBMIT): Pg 492-493, #2, 6, 8, 19*b*