Math 3413.001: Physical Mathematics I

Homework 8, due March 26 (Thursday)

Lecture 17 (Mar 10) Due date 03/26/2020 : Section 7.6

1. Solve the initial value problem

$$x'' + 3x' + 2x = 7u_{11}(t) - 5\delta(t - 2), \qquad x(0) = 2, \quad x'(0) = 1.$$

2. This problem considers a mass m, initially at rest at the origin, that receives an impulse p at time $t = 0$.

   (a) Find the solution $x_\epsilon(t)$ of the initial value problem

   $$mx''(t) = pd_{0,\epsilon}(t), \qquad x(0) = 0, \quad x'(0) = 0,$$

   where the function $d_{0,\epsilon}(t)$ is defined on page 485 of the book.

   (b) Show that $\lim\limits_{\epsilon \to 0+} x_\epsilon(t)$ agrees with the solution of the problem

   $$mx''(t) = p\delta(t), \qquad x(0) = 0, \quad x'(0) = 0.$$

   (c) If $x(t)$ is the solution of the IVP from part (b), show that $mx'(t) = p$.

3. In this problem you will give another proof of the fact that, for any $a > 0$, $u_a' = \delta_a$. Solve the IVP

   $$x' = \delta_a(t), \qquad x(0) = 0$$

   by using Laplace transform, and interpret your result.

4. This problem deals with a mass m on a spring (with elastic constant k) that receives an impulse $p_0 = mv_0$ at time $t = 0$; the mass is initially at rest. Show that the IVPs

   $$mx'' + kx = 0, \qquad x(0) = 0, \quad x'(0) = v_0$$

   and

   $$mx'' + kx = p_0\delta(t), \qquad x(0) = 0, \quad x'(0) = 0$$

   have the same solution. Thus the effect of the term $p_0\delta(t)$ is, indeed, to impart to the particle an initial moment p_0.

   Suggested problems from the book (DO NOT SUBMIT): Pg 492-493, #$2, 6, 8, 19b$

Lecture 18 (Mar 12) Due date 03/26/2020 : Section 9.1

1. Let $f(t)$ be a periodic function with period 2π and

$$f(t) = \begin{cases} 0 & \text{if } -\pi < t \leq 0; \\ \pi & \text{if } 0 < t < \pi. \end{cases}$$

   Find the Fourier series of $f(t)$.

2. Let $f(t)$ be a periodic function with period 2π such that $f(t) = t$ for $-\pi < t \leq \pi$. Find the Fourier series of $f(t)$.

3. Find the Fourier series of

$$f(t) = 2 + 3\cos(2t) - 5\sin(t).$$

   Suggested problems from the book (DO NOT SUBMIT): Pg 572, #$1, 3, 9, 11, 14, 20, 26$