MATH 4433 Homework 8 Due 10/30/2014

Section 4.3: Exercises 4, 15. Hints and remarks:

- there is a useful hint in the book for Exercise 15.

Section 4.4: Exercises 5, 7(a,b). Hints and remarks:

- in Exercise 5 you may use the following facts: the sequence in part (a) is a subsequence of the sequence $\left(\left(1+\frac{1}{n}\right)^n\right)_{n=1}^{\infty}$; in parts (b) and (c) notice that

$$\lim_{n\to\infty}\left(1+\tfrac{1}{n}\right)^{2n} = \lim_{n\to\infty}\left[\left(1+\tfrac{1}{n}\right)^n\left(1+\tfrac{1}{n}\right)^n\right] = \left[\lim_{n\to\infty}\left(1+\tfrac{1}{2n}\right)^n\right]\left[\lim_{n\to\infty}\left(1+\tfrac{1}{2n}\right)^n\right] ,$$

$$\lim_{n\to\infty}\left(1+\tfrac{1}{n}\right)^{n-1} = \lim_{n\to\infty}\left[\left(1+\tfrac{1}{n}\right)^n\left(1+\tfrac{1}{n}\right)^{-1}\right] = \left[\lim_{n\to\infty}\left(1+\tfrac{1}{n}\right)^n\right]\left[\lim_{n\to\infty}\left(1+\tfrac{1}{n}\right)^{-1}\right] ;$$

- in Exercise 7(a,b), you can refer to theorems from the text (please give their exact number).

Section 5.1: Exercises 4, 7(a,b), 8(a), 10(a), 18. Hints and remarks:

- before solving Exercise 4, read Example 5.1.6 and do Practice 5.1.7;
- in Exercise 7(a) you are allowed to use *only* the definition of a limit; the result in Exercise 7(b) follows very easily from the result in part (a);
- in Exercise 8(a), you are allowed to use *only* the definition of a limit; the result follows very easily from the inequality from Exercise 3.2/6(a);
- in Exercise 10(a), you are allowed to use *only* the definition of a limit; one strategy is to assume that $\lim_{x\to c} f(x) = L_1$ and $\lim_{x\to c} f(x) = L_2$, then use the definition of a limit to show that, for an arbitrary $\varepsilon > 0$, $|L_1 - L_2| < \varepsilon$, which by the result of Exercise 3.2/6(c) implies that $L_1 = L_2$;
- in Exercise 18 set, say, $\varepsilon = 1$ in the definition of limit of a function, and then apply reasoning similar to the one in the proof of Theorem 4.1.13; assume that $c \in D$.

Additional problem.

Let $\lambda > 0$ and (s_n) and (t_n) be sequences that satisfy $\lambda < s_n$ $\forall n \in \mathbb{N}$, and $\lim t_n = +\infty$.

(a) Prove that $\lim(s_n t_n) = +\infty$ by using only the definition of a limit of a sequence.

(b) Show (by constructing a counterexample) that the fact proved in part (a) would not hold if the condition $\lambda < s_n$ is replaced with $0 < s_n \; \forall n \in \mathbb{N}$.

Food for Thought:

- Sec. **4.3**, exercises 10, 18.
- Sec. **4.4**, exercises 1(a,b,e), 2(a), 10 (see Example 4.4.5).