MATH 2443 **Homework 9 – Additional Problem** **Summer 2012**

Problem.

Let $\mathbf{F}$ be a vector field on $\mathbb{R}^3$ defined as follows:

$$\mathbf{F}(\mathbf{r}) = \mathbf{F}(x, y, z) = 2xy^2\mathrm{e}^{3z}\,\mathbf{i} + (2x^2y\mathrm{e}^{3z} + z^5\cos y)\,\mathbf{j} + (3x^2y^2\mathrm{e}^{3z} + 5z^4\sin y + 7)\,\mathbf{k}\ .$$

(a) Show that $\mathbf{F}$ is a conservative vector field. Please explain clearly how you did it.
Hint: You used *two* properties of $\mathbf{F}$. Which ones?

(b) Find a potential function $f(\mathbf{r})$ of the vector field $\mathbf{F}$, i.e., a function f such that $\mathbf{F} = \nabla f$.

(c) Find the value of the integral $\int_C \mathbf{F}\cdot\mathrm{d}\mathbf{r}$, where C is the segment of a straight line starting at $(0,0,0)$ and ending at $(3, \frac{\pi}{2}, 2)$.