

Sec. 8.1: problems 6, 13, 22.

Sec. 8.2: problem 33 – solve part (a) completely; for part (b) just show that for integer α one indeed obtains polynomials, and find the explicit expressions for $H_0(x)$, $H_1(x)$, $H_2(x)$, and $H_3(x)$. Hermite polynomials occur in solving the harmonic oscillator problem in quantum mechanics.

Additional problem 1. For a smooth function f of one variable, the Taylor expansion around x_0 is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(x_0) (x - x_0)^n .$$

For a smooth function u of two variables, the Taylor expansion around (x_0, y_0) is given by

$$u(x, y) = \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{k=0}^n \binom{n}{k} \left. \frac{\partial^n u}{\partial x^k \partial y^{n-k}} \right|_{(x_0, y_0)} (x - x_0)^k (y - y_0)^{n-k} ,$$

where, by definition, $0! = 1$ and $\binom{0}{0} = 1$, and $\binom{n}{k}$ (“ n choose k ”) are the binomial coefficients,

$$\binom{n}{k} = \frac{n(n-1)(n-2) \cdots (n-k+1)}{k!} .$$

Let $u(x, y) = (1 + y)^2 + \sin(x + 5y)$. Find the Taylor expansion of $u(x, y)$ around the point $(x_0, y_0) = (\frac{\pi}{2}, 0)$ up to (and including) the terms with $n = 2$. Here is the answer you have to obtain (you have to show your computations in detail):

$$u(x, y) = 2 + 2y - \frac{1}{2} \left(x - \frac{\pi}{2}\right)^2 - 5 \left(x - \frac{\pi}{2}\right) y - \frac{23}{2} y^2 + (\text{higher order terms}) .$$

Additional problem 2. If $u(x, y)$ is a smooth function of two variables, find the following expressions:

$$(a) \quad \frac{d}{dt} u(t, \sin(3t)) ; \quad (b) \quad \frac{d}{dt} u(e^t, \sin(3t)) ; \quad (c) \quad \frac{d^2}{dt^2} u(e^t, \sin(3t)) .$$

Additional problem 3. Let $f(x)$ be a smooth function of one variable. Find the following expressions (where x and y are independent variables):

$$(a) \quad \frac{d}{dt} f(t^3) ; \quad (b) \quad \frac{\partial}{\partial x} f(5x - 3y) ; \quad (c) \quad \frac{\partial}{\partial y} f(5x - 3y) ;$$

$$(d) \quad \left. \frac{\partial}{\partial y} f(5x - 3y) \right|_{(x,y)=(2,7)} .$$