Math 3413.001: Physical Mathematics I

Homework 9, due April 4 (Saturday)

Lecture 19 (Mar 24) Due date 04/02/2020 : Section 9.2

1. Find the Fourier series of the periodic function defined by

$$f(t) = \begin{cases} 0 & \text{if } -2 < t < -1; \\ 1 & \text{if } -1 < t < 1; \\ 0 & \text{if } 1 < t < 2. \end{cases} \qquad f(t+4) = f(t) \text{ for all } t.$$

2. Find the Fourier series of the periodic function defined by

$$f(t) = \begin{cases} 1-t & \text{if } -1 < t < 1; \\ 0 & \text{if } -2 < t < -1 \text{ or } 1 < t < 2; \end{cases} \qquad \text{and} \quad f(t+4) = f(t) \text{ for all } t.$$

3. It can be shown that the Fourier series of $f(t) = t^4$ for $0 < t < 2\pi$ is

$$f(t) = t^4 = \frac{16\pi^4}{5} + 16\sum_{n=1}^{\infty}\left(\frac{2\pi^2}{n^2} - \frac{3}{n^4}\right)\cos(nt) + 16\pi\sum_{n=1}^{\infty}\left(\frac{3}{n^3} - \frac{\pi^2}{n}\right)\sin(nt).$$

   Plug in $t = 0$ and $t = \pi$ to obtain the following formulas

$$\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{\pi^4}{90}, \qquad \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^4} = \frac{7\pi^4}{720}.$$

   Hint: For $t = 0$, you will have to find the average value of the left-hand and right-hand limit of $f(t)$ since it is not continuous at $t = 0$. Also, you will have to use the values of $\sum 1/n^2$ and $\sum(-1)^n/n^2$ obtained in the lecture.

Suggested problems from the book (DO NOT SUBMIT): Pg 578, #3, 7, 9, 13, 17