Math 3413.001: Physical Mathematics I

Homework 9, due April 4 (Saturday)

Lecture 19 (Mar 24) Due date 04/02/2020 : Section 9.2

1. Find the Fourier series of the periodic function defined by

$$f(t) = \begin{cases} 0 & \text{if } -2 < t < -1; \\ 1 & \text{if } -1 < t < 1; \\ 0 & \text{if } 1 < t < 2. \end{cases} \qquad f(t+4) = f(t) \text{ for all } t.$$

2. Find the Fourier series of the periodic function defined by

$$f(t) = \begin{cases} 1-t & \text{if } -1 < t < 1; \\ 0 & \text{if } -2 < t < -1 \text{ or } 1 < t < 2; \end{cases} \qquad \text{and} \quad f(t+4) = f(t) \text{ for all } t.$$

3. It can be shown that the Fourier series of $f(t) = t^4$ for $0 < t < 2\pi$ is

$$f(t) = t^4 = \frac{16\pi^4}{5} + 16\sum_{n=1}^{\infty}\left(\frac{2\pi^2}{n^2} - \frac{3}{n^4}\right)\cos(nt) + 16\pi\sum_{n=1}^{\infty}\left(\frac{3}{n^3} - \frac{\pi^2}{n}\right)\sin(nt).$$

Plug in $t = 0$ and $t = \pi$ to obtain the following formulas

$$\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{\pi^4}{90}, \qquad \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^4} = \frac{7\pi^4}{720}.$$

Hint: For $t = 0$, you will have to find the average value of the left-hand and right-hand limit of $f(t)$ since it is not continuous at $t = 0$. Also, you will have to use the values of $\sum 1/n^2$ and $\sum(-1)^n/n^2$ obtained in the lecture.

Suggested problems from the book (DO NOT SUBMIT): Pg 578, $\#3, 7, 9, 13, 17$

Lecture 20 (Mar 26) Due date 04/02/2020 : Section 9.3

1. We have three functions that are periodic with period 2, and on the interval $(-1, 1)$, they are defined by the following formulas

$$f_1(t) = t, \qquad f_2(t) = |t|, \qquad f_3(t) = t + |t|.$$

Exactly one of the above functions is odd, one is even, and one neither. Determine (and explain) which is which. Then find the Fourier series of the even and odd function.

2. Let

$$f(t) = e^t \qquad \text{for } 0 < t < 1.$$

Draw the graphs of the even and odd extensions of $f(t)$.

3. Find the Fourier sine series and the Fourier cosine series of the function in problem 2. *Hint*: You can use the integration formulas

$$\int e^u\cos(\beta u)du = \frac{e^u\left[\cos(\beta u) + \beta\sin(\beta u)\right]}{1+\beta^2} + C, \qquad \int e^u\sin(\beta u)du = \frac{e^u\left[-\beta\cos(\beta u) + \sin(\beta u)\right]}{1+\beta^2} + C$$

Suggested problems from the book (DO NOT SUBMIT): Pg 589-590, $\#1, 5, 7$