```
(*  Illustration of the Newton's method for solving nonlinear equations  *)

(*  First solve the equation f(x)=0 for f(x)=x^2-5 in order to find sqrt(5)  *)

(*  Define the function and find its derivative  *)

In[1]:= f[x_] = x^2 - 5
```

Out[1]= $-5 + x^2$

```
In[2]:= D[f[x], x]
```

Out[2]= $2\,x$

```
(*  Define the initial guess and compute the initial value of the error  *)

In[3]:= x1 = 2
```

Out[3]= 2

```
In[4]:= e1 = Abs[x1 - Sqrt[5]];

In[5]:= N[e1, 100]

Out[5]= 0.2360679774997896964091736687312762354406183596115257242708972454105209256378048\
        994144144083787822750

(*  Start iterating; at each step compute the (absolute)
 error as the absolute value of the difference between the
 current and the true value and print 100 digits of the error  *)

In[6]:= x2 = x1 - (x1^2 - 5) / (2 * x1)
```

Out[6]= $\frac{9}{4}$

```
In[7]:= e2 = Abs[x2 - Sqrt[5]];

In[8]:= N[e2, 100]

Out[8]= 0.0139320225002103035908263312687237645593816403884742757291027545894790743621951\
        0058558559162121772503

In[9]:= x3 = x2 - (x2^2 - 5) / (2 * x2)
```

Out[9]= $\frac{161}{72}$

```
In[10]:= e3 = Abs[x3 - Sqrt[5]];

In[11]:= N[e3, 100]

Out[11]= 0.0000431336113214147019374423798348756704927514995853868402138657005901854733062\
         1169669670273232883614160

In[12]:= x4 = x3 - (x3^2 - 5) / (2 * x3)
```

Out[12]= $\frac{51\,841}{23\,184}$

In[13]:= `e4 = Abs[x4 - Sqrt[5]];`

In[14]:= `N[e4, 100]`

Out[14]= $4.1601430635135083092365820210251483569558262631571421076615264234011090458834611612087265946803495 70 \times 10^{-10}$

In[15]:= `x5 = x4 - (x4^2 - 5) / (2 * x4)`

Out[15]= $\frac{5\,374\,978\,561}{2\,403\,763\,488}$

In[16]:= `e5 = Abs[x5 - Sqrt[5]];`

In[17]:= `N[e5, 100]`

Out[17]= $3.869915959583412293087879493715007290470241990896494883937660938467423141005871436496331907479975678 \times 10^{-20}$

In[18]:= `x6 = x5 - (x5^2 - 5) / (2 * x5)`

Out[18]= $\frac{57\,780\,789\,062\,419\,261\,441}{25\,840\,354\,427\,429\,161\,536}$

In[19]:= `e6 = Abs[x6 - Sqrt[5]];`

In[20]:= `N[e6, 100]`

Out[20]= $3.348791200655663272228308316782282108003483992181542389564978782979739269168833971559384790745890207 \times 10^{-40}$

In[21]:= `x7 = x6 - (x6^2 - 5) / (2 * x6)`

Out[21]= $\frac{6\,677\,239\,169\,351\,578\,707\,225\,356\,193\,679\,818\,792\,961}{2\,986\,152\,136\,938\,872\,067\,784\,669\,198\,846\,010\,266\,752}$

In[22]:= `e7 = Abs[x7 - Sqrt[5]];`

N::meprec : Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating $\frac{6677239169351578707225356193679818792961}{2986152136938872067784669198846010266752} - \sqrt{5}$. ≫

In[23]:= `N[e7, 100]`

N::meprec : Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating $\text{Abs}\left[\frac{6677239169351578707225356193679818792961}{2986152136938872067784669198846010266752} - \sqrt{5}\right]$. ≫

Out[23]= $2.507616632954051933422778296042349185183385554328231434526407442904981 \times 10^{-80}$

```
(*  Here is the same procedure
 (using 10000 digits of the numbers in each computation) with overloading
 the value of x by the current value and printing the error at each step  *)
```

In[24]:=
```
f[x_] := x^2 - 5;
x = N[2, 10000];
For[i = 1, i ≤ 12, i++,
 {x = x - f[x] / f'[x],
  error = Abs[x - Sqrt[5]],
  Print[i, "    ", 1.0 * error]
 }
]
```

1   0.013932

2   0.0000431336

3   4.16014×10^{-10}

4   3.86992×10^{-20}

5   3.34879×10^{-40}

6   2.50762×10^{-80}

7   1.40607×10^{-160}

8    $4.420786831021170 \times 10^{-321}$

9    $4.370027298361066 \times 10^{-642}$

10    $4.270250005944356 \times 10^{-1284}$

11    $4.077477808536231 \times 10^{-2568}$

12    $3.717647550611415 \times 10^{-5136}$