On Problem 1.8(a) of Ross

- Using elementary algebra: complete the square:

$$x^2 - x - 1 = x^2 - 2\cdot x \cdot \tfrac{1}{2} + \tfrac{1}{4} - \tfrac{5}{4} = \left(x - \tfrac{1}{2}\right)^2 - \tfrac{5}{4}$$

which is strictly positive if

$$\left|x - \tfrac{1}{2}\right| > \sqrt{\tfrac{5}{4}},$$

i.e., if

$$-0.618\ldots = \tfrac{1}{2} - \sqrt{\tfrac{5}{4}} < x < \tfrac{1}{2} + \sqrt{\tfrac{5}{4}} = 1.618\ldots,$$

which includes the set $\{2, 3, 4, 5, \ldots\}$.

- Using Calculus: the roots of the equation

$$f(x) := x^2 - x - 1 = 0$$

are $x_1 = \frac{1}{2} - \sqrt{\frac{5}{4}}$ and $x_2 = \frac{1}{2} + \sqrt{\frac{5}{4}} \approx 1.618$, and

$$f'(x) = 2x - 1$$

which is strictly positive if $x > \frac{1}{2}$. Therefore, $f(x_2) = 0$ and $f'(x) > 0$ on the interval $[x_2, \infty)$ (the latter means that f is strictly increasing on $[x_2, \infty)$), so

$$0 = f(x_2) < f(x) \qquad \forall x \in [2, \infty).$$