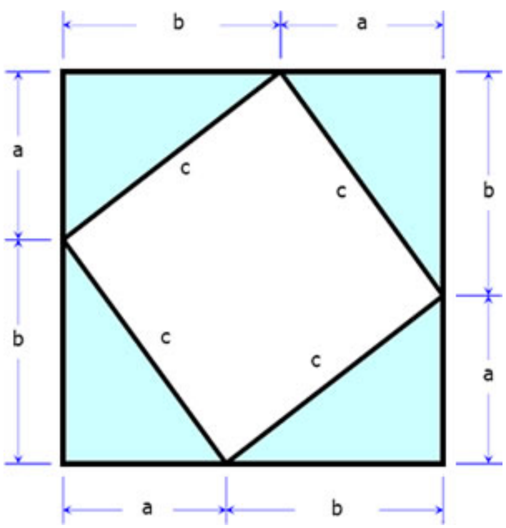
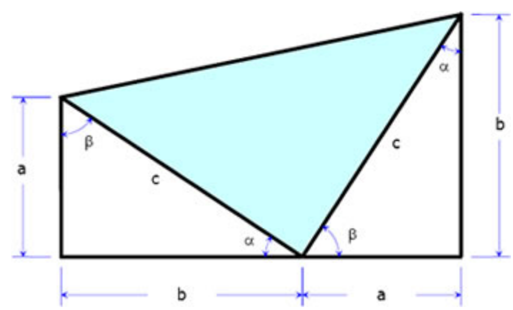
Proofs of Pythagorean Theorem

1 Proof by Pythagoras (ca. 570 BC–ca. 495 BC) (on the left) and by US president James Garfield (1831–1881) (on the right)

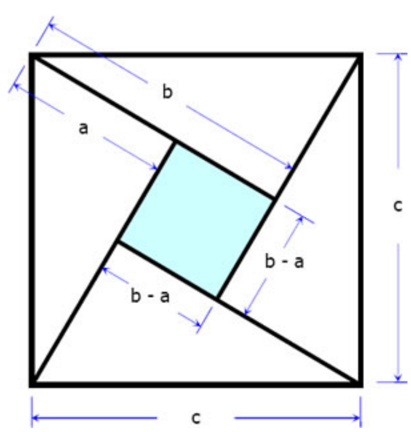
Proof by Pythagoras: in the figure on the left, the area of the large square (which is equal to $(a+b)^2$) is equal to the sum of the areas of the four triangles ($\frac{1}{2}ab$ each triangle) and the area of the small square (c^2):

$$(a+b)^2 = 4\left(\frac{1}{2}ab\right) + c^2 \quad \Rightarrow \quad a^2 + 2ab + b^2 = 2ab + c^2 \quad \Rightarrow \quad a^2 + b^2 = c^2 \ .$$



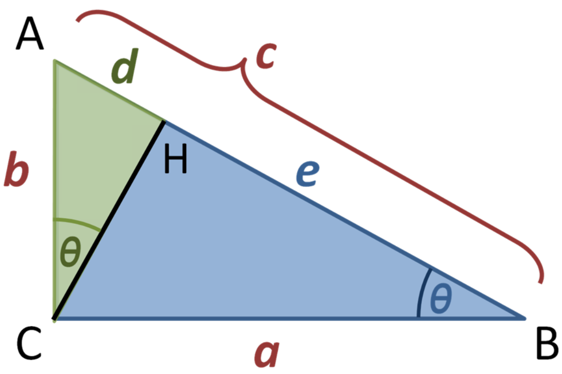


2 Proof by Bhaskara (1114–1185)



3 Proof by similar triangles

Let CH be the perpendicular from C to the side AB in the right triangle $\triangle$ABC.



Observation 1: $\triangle\mathrm{ABC} \sim \triangle\mathrm{CBH}$, therefore $\frac{\mathrm{AB}}{\mathrm{BC}} = \frac{\mathrm{CB}}{\mathrm{CH}}$, i.e., $\frac{c}{a} = \frac{a}{e}$, hence $e = \frac{a^2}{c}$. (1)

Observation 2: $\triangle\mathrm{ABC} \sim \triangle\mathrm{ACH}$, therefore $\frac{\mathrm{AB}}{\mathrm{AC}} = \frac{\mathrm{AC}}{\mathrm{AH}}$, i.e., $\frac{c}{b} = \frac{b}{d}$, hence $d = \frac{b^2}{c}$. (2)

Finally, $\mathrm{AB} = \mathrm{BH} + \mathrm{AH}$, i.e, $c = e + d$. Using (1) and (2), we rewrite this as $c = \frac{a^2}{c} + \frac{b^2}{c}$, which is equivalent to $c^2 = a^2 + b^2$.

References

The book

Elisha Scott Loomis, *The Pythagorean Proposition: Its Demonstrations Analyzed and Classified, and Bibliography of Sources for Data of the Four Kinds of "Proofs"*, Second edition, 1940, available at `http://files.eric.ed.gov/fulltext/ED037335.pdf`

contains 370 proofs of the Pythagorean Theorem.