MATH 4103 Quiz 10 Spring 2016

Name: _____

Problem 1. [1+1+1 points] Consider the function

$$f(z) = \frac{1}{1-z} \, . \tag{1}$$

In this problem you will obtain the Taylor expansion of $f(z)$ about $z_0 = i$:

$$f(z) = \sum_{n=0}^{\infty} a_n \, (z-i)^n \, . \tag{2}$$

(a) Where is the function $f(z)$ from (1) analytic? Why?

(b) Without doing any calculations, tell me what is the largest open disk in which the Taylor expansion (2) about $z_0 = i$ of the function $f(z)$ from (1) is going to converge. Explain briefly how you came to this conclusion. (*Hint:* A picture may be helpful for both you and me...)

(c) Find the desired Taylor expansion (2) of $f(z)$ from (1).