Mock Quiz 6

Problem 1. If f is a differentiable function, find $\frac{d}{dx}\, e^{f(x)}$ and $\frac{d}{dx}\, f\left(e^x\right)$.

Answer: $\frac{d}{dx}\, e^{f(x)} = e^{f(x)} f'(x)$, $\frac{d}{dx}\, f\left(e^x\right) = f'\left(e^x\right)\, e^x$.

Problem 2. If $F(x) = e^{t \sin 2t}$, find $F'(x)$ and $F'(\frac{\pi}{6})$.

Problem 3. Let C be a curve in the (x, y)-plane defined implicitly by the equation

$$x\, e^y + e^x\, y^2 = 2\ .$$

(a) Show that the point $(2, 0)$ belongs to C.

Answer: $2 \cdot e^0 + e^2 \cdot 0^2 = 2$ ✓

(b) Think of y as a function of x and use implicit differentiation to find $\frac{dy}{dx}$.

Answer: $e^{y(x)} + x e^{y(x)} y'(x) + e^x y(x)^2 + e^x 2y(x) y'(x) = 0$,
express $y'(x)$: $\frac{dy}{dx} = y'(x) = -\frac{e^y + e^x y^2}{x e^y + 2e^x y}$

(c) Find the slope of the tangent line to C at the point $(2, 0)$.

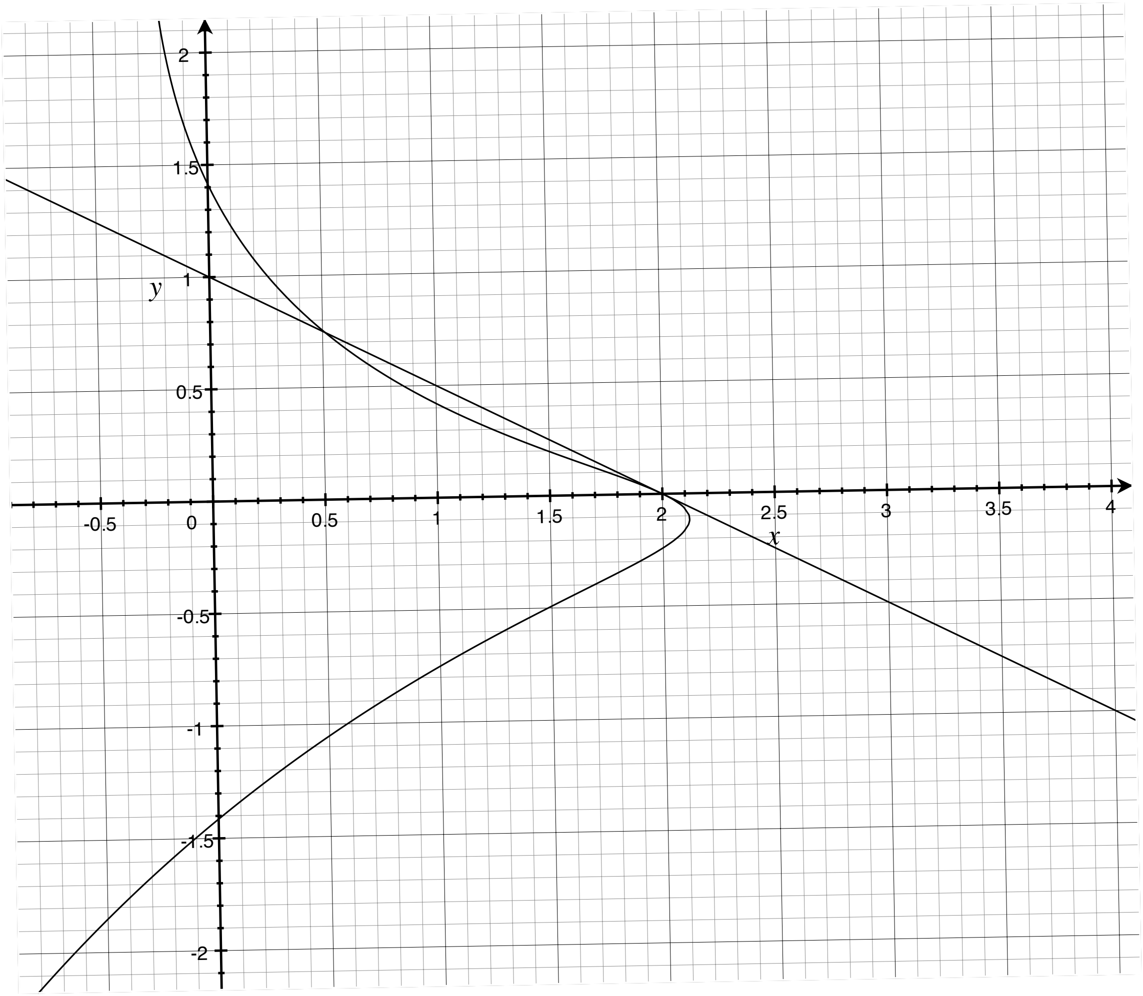
Answer: $\left.\frac{dy}{dx}\right|_{(2,0)} = -\left.\frac{e^y + e^x y^2}{x e^y + 2e^x y}\right|_{(2,0)} = -\frac{1}{2}$

(d) Write down the equation of the tangent line to C at $(2, 0)$.

Answer: $\frac{y-0}{x-2} = -\frac{1}{2}$, i.e., $y = 1 - \frac{x}{2}$; see the curve C and the tangent line to C at $(2, 0)$ in the graph below.

(e) Think of x as a function of y and use implicit differentiation to find $\frac{dx}{dy}$ and the numerical value of $\frac{dx}{dy}$ at $(x, y) = (2, 0)$.

Answer: $\frac{dx}{dy} = -\frac{x e^y + 2e^x y}{e^y + e^x y^2}$, $\left.\frac{dx}{dy}\right|_{(x,y)=(2,0)} = -2.$


2
1.5
1
y
0.5
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
x
-0.5
-1
-1.5
-2