Rabbits vs sheep

$$\left|\begin{array}{l} x(t) = \text{population of rabbits} \\ y(t) = \text{population of sheep} \end{array}\right.$$

$$\left|\begin{array}{l} x' = x(3-x-2y) \\ y' = y(2-x-y) \end{array}\right. \quad : \quad \underline{x}' = \underline{f}(\underline{x})$$

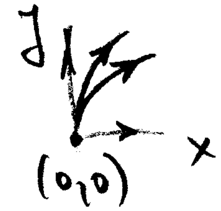
Fixed points: $(0,0)$, $(0,2)$, $(3,0)$, $(1,1)$.

Jacobian matrix: $D\underline{f}(\underline{x}) = \begin{pmatrix} 3-2x-2y & -2x \\ -y & 2-x-2y \end{pmatrix}$

- $\underline{(0,0)}$: $D\underline{f}(0,0) = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$, $\lambda_1 = 3$, $\underline{v}^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

  $\lambda_2 = 2$, $\underline{v}^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

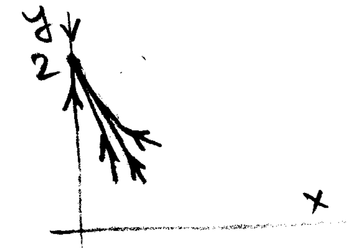
  unstable node



- $\underline{(0,2)}$: $D\underline{f}(0,2) = \begin{pmatrix} -1 & 0 \\ -2 & 2 \end{pmatrix}$, $\lambda_1 = -1$, $\underline{v}^{(1)} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

  $\lambda_2 = -2$, $\underline{v}^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

  stable node



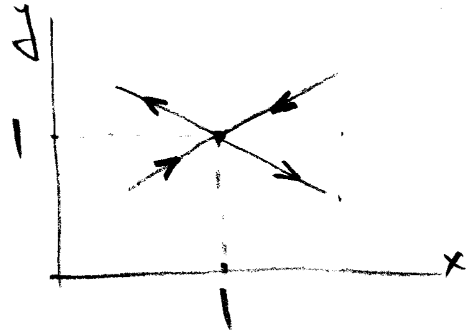
- $\underline{(3,0)}$: $D\underline{f}(3,0) = \begin{pmatrix} -3 & -6 \\ 0 & -1 \end{pmatrix}$; $\lambda_1 = -3$, $\underline{v}^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

  $\lambda_2 = -1$, $\underline{v}^{(2)} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

  stable node

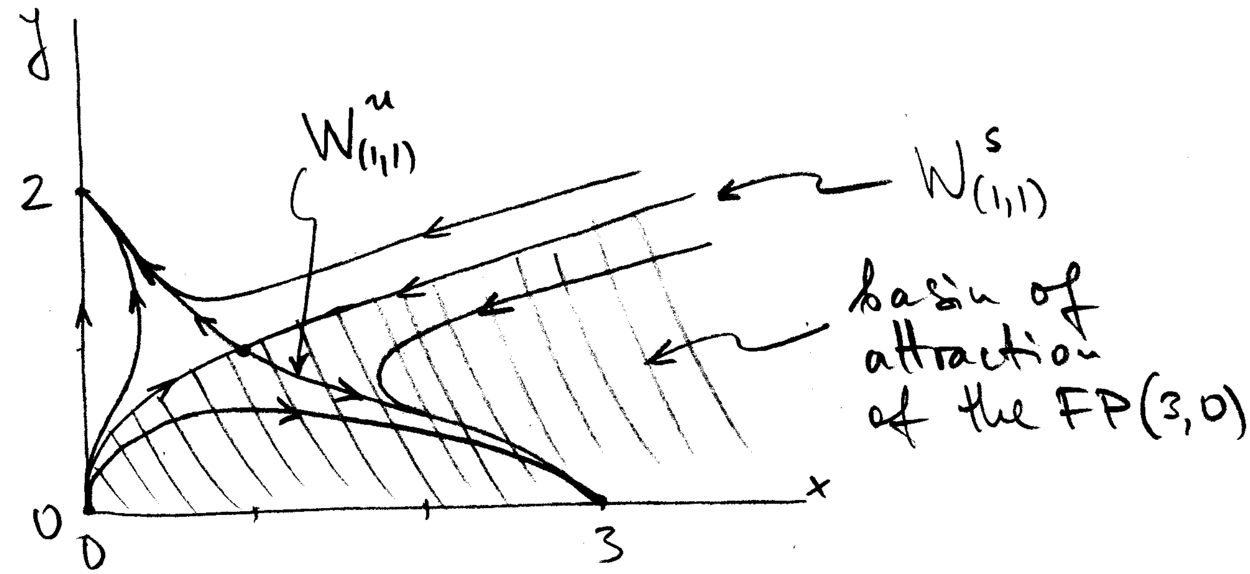
• $\underline{(1,1)}$: $D\underline{f}(1,1) = \begin{pmatrix} -1 & -2 \\ -1 & -1 \end{pmatrix}$, $\lambda_{1,2} = -1 \pm \sqrt{2}$

$\underline{v}^{(1,2)} = \begin{pmatrix} \sqrt{2} \\ \mp 1 \end{pmatrix}$



$\lambda_2 < 0 < \lambda_1$
saddle

<u>Putting everything together</u>:



Stable and unstable manifolds of the hyperbolic FP (1,1):

$$W^s_{(1,1)} := \{ \underline{x} \in \mathbb{R}^2 : \lim_{t \to \infty} \underline{\varphi}_t(\underline{x}) = (1,1) \},$$

$$W^u_{(1,1)} := \{ \underline{x} \in \mathbb{R}^2 : \lim_{t \to -\infty} \underline{\varphi}_t(\underline{x}) = (1,1) \}$$