**10.** Fill in the blanks in the proof of the following theorem.

**THEOREM:** $A \subseteq B$ iff $A \cup B = B$.

**Proof:** Suppose that $A \subseteq B$. If $x \in A \cup B$, then $x \in A$ or $x \in$ __________. Since $A \subseteq B$, in either case we have $x \in B$. Thus __________ $\subseteq$ __________. On the other hand, if $x \in$ __________, then $x \in A \cup B$, so __________ $\subseteq$ __________. Hence $A \cup B = B$.

Conversely, suppose that $A \cup B = B$. If $x \in A$, then $x \in$ __________. But $A \cup B = B$, so $x \in$ __________. Thus __________ $\subseteq$ __________. ♦

**11.** Fill in the blanks in the proof of the following theorem.

**THEOREM:** $A \subseteq B$ iff $A \cap B = A$.

**Proof:** Suppose that $A \subseteq B$. If $x \in A \cap B$, then clearly $x \in A$. Thus $A \cap B \subseteq A$. On the other hand, ________________________________________________________________________.

Thus $A \subseteq A \cap B$, and we conclude that $A \cap B = A$.

Conversely, suppose that $A \cap B = A$. If $x \in A$, then ________________________________________________________________________.

Thus $A \subseteq B$. ♦