10. Fill in the blanks in the proof of the following theorem.

THEOREM: $A \subseteq B$ iff $A \cup B = B$.

Proof: Suppose that $A \subseteq B$. If $x \in A \cup B$, then $x \in A$ or $x \in$ B. Since $A \subseteq B$, in either case we have $x \in B$. Thus $A \cup B$ $\subseteq$ B. On the other hand, if $x \in$ B, then $x \in A \cup B$, so B $\subseteq$ $A \cup B$. Hence $A \cup B = B$.

Conversely, suppose that $A \cup B = B$. If $x \in A$, then $x \in$ $A \cup B$. But $A \cup B = B$, so $x \in$ B. Thus A $\subseteq$ B. ♦

11. Fill in the blanks in the proof of the following theorem.

THEOREM: $A \subseteq B$ iff $A \cap B = A$.

Proof: Suppose that $A \subseteq B$. If $x \in A \cap B$, then clearly $x \in A$. Thus $A \cap B \subseteq A$. On the other hand, if $x \in A$, then since $A \subseteq B$, $x \in B$ and, by def., $(x \in A$ & $x \in B) \Leftrightarrow x \in A \cap B$. Thus $A \subseteq A \cap B$, and we conclude that $A \cap B = A$.

Conversely, suppose that $A \cap B = A$. If $x \in A$, then since $A = A \cap B$, $x \in A \cap B$. But $A \cap B \subseteq B \Rightarrow x \in B$. Thus $A \subseteq B$. ♦