Problem 1. Let d and t be the following statements:

d : You drive over 65 mph.

t : You get a speeding ticket.

["mph" stands for "miles per hour"] Write the following statements using d and t and logical connectives ($\sim$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$).

(example) You do not drive over 65 mph. $\sim d$

(a) You drive over 65 mph, but you do not get a speeding ticket. $d \wedge \sim t$

(b) You get a speeding ticket if you drive over 65 mph. $d \Rightarrow t$

(c) Driving over 65 mph is a sufficient condition for getting a speeding ticket. $d \Rightarrow t$

(d) You get a speeding ticket iff you drive over 65 mph. $t \Leftrightarrow d$

(e) Whenever you get a speeding ticket, you are driving over 65 mph. $t \Rightarrow d$

(f) Driving over 65 mph is a necessary condition for getting a speeding ticket. $t \Rightarrow d$

(g) Write down the contrapositive of the statement

If you drive over 65 mph, you get a speeding ticket.

Contrapositive: If you do not get a speeding ticket, then you do not drive over 65 mph.

Problem 2. In each of the following cases, determine whether a statement is true or false. If it is true, explain briefly; it is false, give a counterexample. Always think of x and y as real numbers.

(a) $\forall x \, \forall y \,, \; x^2 + y^2 > 0$ False: For $x = 0$ and $y = 0$, $x^2 + y^2 = 0 \not> 0$.

(b) $\exists x$ s.t. $\forall y \,, \; x < y^2$ True: Since $y^2 \geq 0 \; \forall y$, we can take, $x = -5$, then $\forall y$, $-5 = x < y^2$.

Problem 3. Let the function $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(n) = n^2$.

(a) Determine $f(C)$ if $C = \{-2, 0, 1, 2\}$. $f(C) = \{0, 1, 4\}$

(b) Determine $f^{-1}(D)$ if $D = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. $f^{-1}(D) = \{-3, -2, -1, 0, 1, 2, 3\}$

(c) Is f surjective (i.e., onto)? Explain briefly. No, because there there is no $n \in \mathbb{Z}$ so that $f(n)$ is negative (hence, $\operatorname{rng} f = f(\mathbb{Z}) = \{0, 1, 2, 3, \ldots\}$).

(d) Is f bijective? Explain briefly why or why not. No, because f is not surjective. (Incidentally, f is also not injective.)