Trace-determinant plane

$$\left|\begin{array}{l} x' = ax + by \\ y' = cx + dy \end{array}\right. , \quad \underline{x}' = A\underline{x}, \quad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$0 = \det(A - \lambda I) = \lambda^2 - (\operatorname{tr} A)\lambda + \det A$$

$$\tau := \operatorname{tr} A = a + d, \quad \Delta := \det A = ad - bc$$

$$\lambda^2 - \tau\lambda + \Delta = 0$$

$$\Rightarrow \quad \lambda_{1,2} = \frac{1}{2}\left(\tau \pm \sqrt{\tau^2 - 4\Delta}\right)$$

Observations:

- $\Delta < 0 \rightarrow \lambda_1$ and λ_2 have different signs $\Longrightarrow$ <u>saddle points</u>
- $\Delta > 0,\ \tau^2 - 4\Delta > 0 \Rightarrow \lambda_1, \lambda_2 \in \mathbb{R}$, same sign $\Rightarrow$ <u>nodes</u>
- $\Delta > 0,\ \tau^2 - 4\Delta < 0 \Rightarrow \lambda_1 = \overline{\lambda_2} \notin \mathbb{R}$
  $\Rightarrow$ <u>spirals</u> ($\mathrm{Re}\,\lambda_{1,2} \neq 0$), <u>centers</u> ($\mathrm{Re}\,\lambda_{1,2} = 0$)