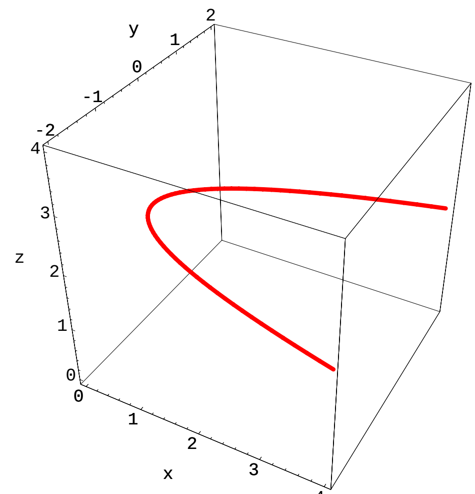
CALCULUS III FALL 1999
HOMEWORK 13 – ANSWERS

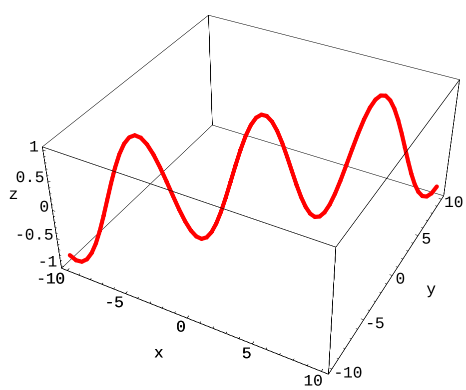
§11.7 Questions 8,12,30,40,44,52,56,74; §11.8 Questions 4,6,10

8.



t increases in the direction of increasing y.

12.



t increases in the direction of increasing x and y.

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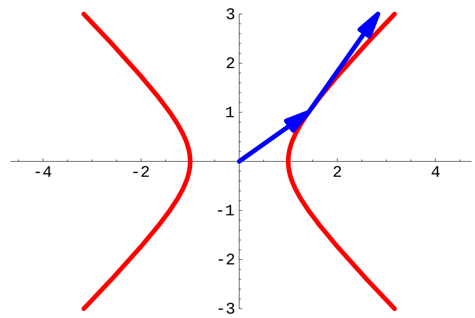
30. The domain comprises all values of t except -1. Differentiation gives

$$\mathbf{r}'(t) = (2t+1)e^{2t}\mathbf{i} + \frac{2}{(t+1)^2}\mathbf{j} + \frac{1}{t^2+1}\mathbf{k}.$$

40.



Note that this is the graph of a hyperbola since $1 + \tan^2 t = \sec^2 t$.

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52. First we have to find $s,\ t$ such that $t = 3 - s,\ 1 - t = s - 2,\ 3 + t^2 = s^2$. It is easy to check that this implies $s = 2,\ t = 1$. Consequently the point of intersection is $(1, 0, 4)$. We next find the angle between their tangent vectors at this point. We have

$$\mathbf{r}_1'(t) = \mathbf{i} - \mathbf{j} + 2t\mathbf{k} \qquad \mathbf{r}_2'(s) = -\mathbf{i} + \mathbf{j} + 2s\mathbf{k}.$$

Hence $\mathbf{r}_1'(1) = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{r}_2'(2) = -\mathbf{i} + \mathbf{j} + 4\mathbf{k}$. Hence $\mathbf{r}_1'(1).\mathbf{r}_2'(2) = 6$. We also have $|\mathbf{r}_1'(1) = \sqrt{6}|$ and $|\mathbf{r}_2'(2)| = 3\sqrt{2}$. It follows that the required angle is $\cos^{-1}\sqrt{1/3} \simeq 55°$.

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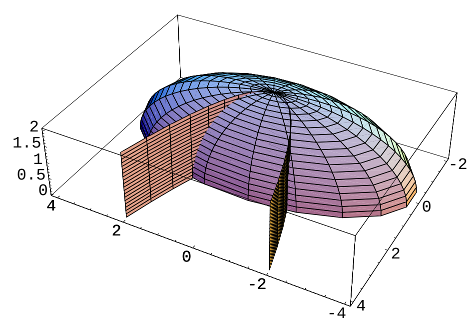
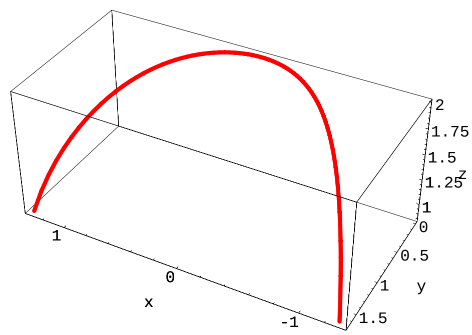
56. The intersection of the surfaces looks like this



The parametric equations of the curve are $x = t,\ y = t^2,\ z = \sqrt{4 - t^4 - t^2/4}$. Its graph is shown below



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74. Note that

$$\frac{d}{dt}\mathbf{r}(t).\mathbf{r}(t) = 2\mathbf{r}(t).\mathbf{r}'(t) = 0.$$

Consequently $\mathbf{r}(t).\mathbf{r}(t)$ is constant which implies $|\mathbf{r}(t)|$ is constant, as required.

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4. We have $\mathbf{r}'(t) = 2t\mathbf{i} + 2\mathbf{j} + (1/t)\mathbf{k}$ and so the required length is

$$\int_1^e \sqrt{4t^2 + 4 + \frac{1}{t^2}}\, dt = \int_1^e \left(\frac{1}{t} + 2t\right)\, dt = \left[\ln t + t^2\right]_1^e = e^2.$$

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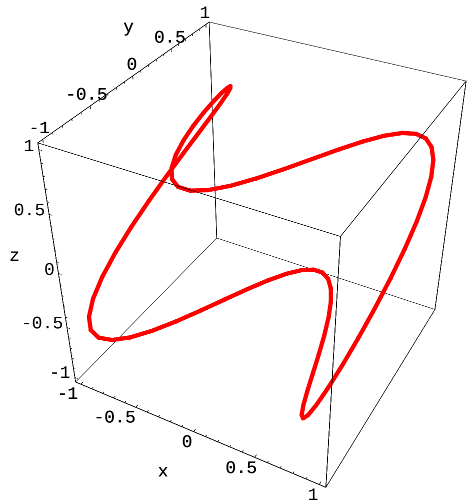
6. The curve is



Its length is

$$\int_0^{2\pi} \sqrt{(-\sin t)^2 + (3\cos 3t)^2 + (\cos t)^2}\, dt = \int_0^{2\pi} \sqrt{1 + 9\cos^2 3t}\, dt \simeq 13.9744$$

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10. We have $\mathbf{r}'(t) = -3\cos^2 t \sin t \mathbf{i} + 3\sin^2 t \cos t \mathbf{j} - 2\sin 2t \mathbf{k}$. Consequently

$$\frac{ds}{dt} = \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t + 4\sin^2 2t} = \frac{5}{2}\sin 2t.$$

It follows that

$$s(t_0) = \int_0^{t_0} \frac{5}{2}\sin 2t\, dt = \frac{5}{4} - \frac{5}{4}\cos t_0 = \frac{5}{2}\sin^2 t.$$

Therefore we have

$$\sin^2 t = \frac{2s}{5} \qquad \cos^2 t = 1 - \frac{2s}{5} \qquad \cos 2t = 1 - \frac{4s}{5}.$$

The required reparameterization is

$$\mathbf{r}(s) = \left(1 - \frac{2s}{5}\right)^{3/2} \mathbf{i} + \left(\frac{2s}{5}\right)^{3/2} \mathbf{j} + \left(1 - \frac{4s}{5}\right) \mathbf{k}.$$