# CALCULUS III  FALL 1999

# HOMEWORK 5 – ANSWERS

**§10.1 Questions 46,52,54,60**

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**46.**

$$a_n = (3^n + 5^n)^{1/n} = 5\left(1 + \left(\frac{3}{5}\right)^n\right)^{1/n} \to 5 \qquad \text{as } n \to \infty.$$

The same idea can be used to show, more generally, that $(a^n + b^n)^{1/n} \to \max\{a, b\}$.

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**52.** $a_{n+1} \leqslant a_n \iff 5^n \leqslant 5^{n+1}$, which is clearly true and so the sequence is decreasing.

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**54.**

$$a_{n+1} - a_n = \frac{3n+7}{2n+7} - \frac{3n+4}{2n+5} = \frac{7}{(2n+7)(2n+5)} > 0,$$

and so the sequence is increasing.

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**60.** We see that $a_{n+1}^2 - a_n^2 = 2 + a_n - a_n^2 = (2 - a_n)(1 + a_n)$. It follows that $a_{n+1} > a_n$ if $-1 < a_n < 2$ (note that we clearly have $a_n \geqslant 0$ for all $n$). Next note that $4 - a_{n+1}^2 = 2 - a_n$ and so, if $a_n < 2$, then $a_{n+1} < 2$. It therefore follows that $a_n < 2$ for all $n$ since $a_1 < 2$. Our previous observations now show that the sequence is increasing and $0 \leqslant a_n < 2 < 3$ for all $n$. It follows that the sequence converges (Theorem 10 of the book). If $\ell$ is its limit then $\ell$ must satisfy $\ell = \sqrt{2 + \ell}$ which gives $(\ell - 2)(\ell + 1) = 0$. Hence $\ell = -1$ or 2. Each term of the sequence is non-negative and so the limit cannot be $-1$. It follows that $a_n \to 2$ as $n \to \infty$.